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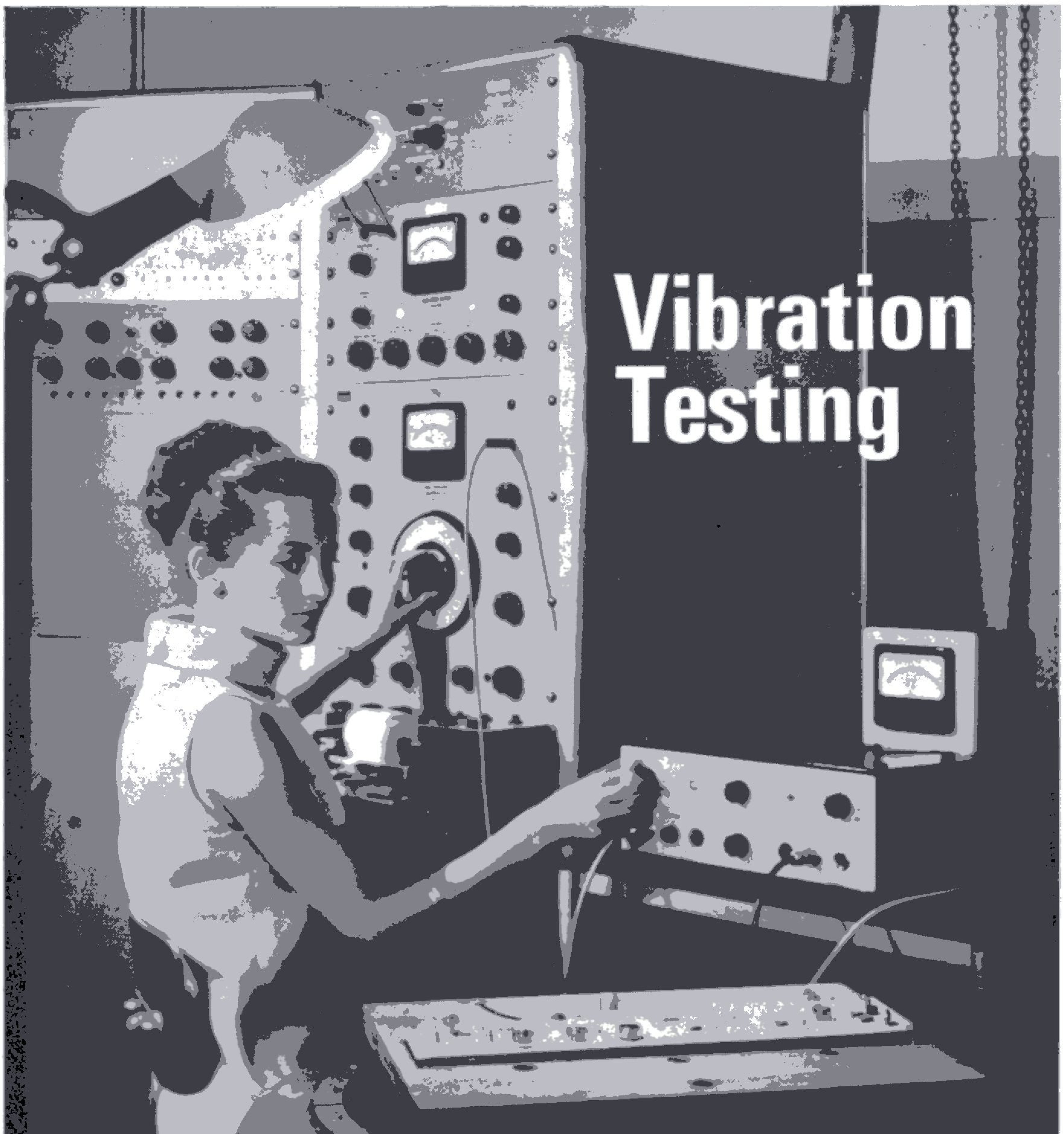

No. 3 1967

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TECHNICAL REVIEW

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Vibration Testing –
The Reasons and The Means

by

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ABSTRACT

Following a brief historical review of the development of vibration testing some details of present day vibration test equipment and techniques are given. Starting with the vibration machine itself the main operating ranges of modern electro-hydraulic and electro-dynamic vibrators are outlined. Then follows a discussion of the most commonly used vibration test techniques,—the sweeping sine wave test,—and the philosophies behind the development of test specifications for this type of testing are mentioned.

The concepts of random vibrations, probability distributions and power spectral densities are subsequently introduced and applied to a second method of vibration testing,–the wide-band random test.

Finally a third, relatively new method of vibration testing,—the sweep random vibration test,–is described and the derivation of appropriate test specifications discussed.

SOMMAIRE

Après un bref aperçu hstorique du développement de la technique des essais aux vibrations, quelques détails sont donnés concernant l'équipement et les techniques d'essai aux vibrations. Partant du générateur de vibrations lui-même, on esquisse les principaux domaines de travail des générateurs électro-hydrauliques et électro-dynamiques.

Vient alors une discussion des techniques d'essai aux vibrations les plus couramment utilisées, à commencer par l'essai sous onde sinusoïdale glissée en fréquence et les considérations à la base de la formulation des spécifications pour ce genre de contrôles.

Ensuite sont introduites les nations de vibrations aléatoires, de distributions de probabilités et de densité spectrales d'énergie. On les applique à une seconde méthode d'essai aux vibrations, qui fait appel aux signaux aléatoires sous large bande.

Finalement une troisième méthode d'éssai aux vibrations, relativement nouvelle, utilisant le balayage en fréquence d'une bande étroite de signaux aléatoires, est décrite et l'on discute de la formulation de spécifications d'essai appropriées.

ZUSAMMENFASSUNG

Nach einem kurzen historischen Rückblick auf die Entwicklung der Vibrationsprüfung werden einige Details über heutige Schwingprüf-Anlagen und -Methoden mitgeteilt. Beginnend mit dem eigentlichen Schwingerreger werden die wichtigsten Ensatzbereche moderner elektrohydraulischer und elektrodynamischer Schwingungserzeuger umrissen. Dann folgt eine Erörterung der gebräuchlichsten Schwingprüfmethode, die Prüfung mit Sinus gleitender Frequenz, — und es werden die Grundanschauungen erwähnt, auf deres Hintergrund Prüfvorschriften für diese Art der Prüfung entwickelt wurden.

Nachfolgend werden die Begriffe stochastische Schwingungen, Wahrscheinlichkeitsverteilungen und spektrale Leistungsdichte eingeführt und auf eine zweite Methode der Schwingprüfung, die Prüfung mit Breitbandrauschen, angewandt.

Schließlich wird eine dritte, relativ neue Methode der Schwingprüfung, die Prüfung mit Schmalbandrauschen gleitender Frequenz, beschrieben und die Ableitung geeigneter Prüfvorschriften diskutiert.

*) Invited tutorial lecture presented in Prague March 15. 1967.

Introduction.

The concept of vibration testing as we know it to-day is a relatively new concept. It originated more or less with the desire to test parts and equipment for use in airplanes prior to a first flight. Even though vibration testing, in a sense, was used by Wöhler some 100 years ago in his experiments on the fatigue of metals, the intensive use of a more general vibration test technique developed during and subsequent to World War II.

At this time not only structural mechanical failures due to vibrations became a problem, but also the use of complicated electronic and electro-mechanical equipment made control systems and communication instrumentation sensitive to the vibrations encountered during mobile operation. Furthermore the speed and maneuvering facilities available in modern vehicles severely increase the vibrations caused by the overall environment and complicate theoretical predictions and estimates of vibration responses.

While in the early days of flight and motoring the main vibrations induced in the structure and control equipment originated from a certain unbalance in the engine, today's vibration damage is almost equally caused by vibrations induced by the overall environment and by vibrations due to engine effects. This, of course, also influences the requirements to be met by laboratory vibration tests and tends to complicate the test equipment.

Some twenty to thirty years ago "laboratory" vibration tests were practically always carried out in the form of resonance search and dwell testing and/or testing at specified fixed frequencies. These fixed frequencies then corresponded to those produced by the engine when operated at a recommended speed. An interesting fact, which becomes evident by looking at vibration test specifications issued over the past three decades, is the increase in upper frequency limit specified for the test. This emphasizes the fact that the main vibrations were produced by the engine itself, and that faster running engines were utilized.

However, about fifteen years ago the dwell testing at certain frequencies started to be complemented by frequency sweep tests, the main reason being, that during operation the frequencies produced by the engines were not fixed but changed with speed. Also, around that time vibrations induced by the overall environment were beginning to become significant. This was further emphasized by the introduction of jet powered flights.

Even though the development of vibration testing techniques quite naturally have been closely cennected to the aircraft and space vehicle field it might be worth mentioning that vibration testing, often supplemented by shock and bump testing is today also used in many other fields. Typical examples are the field of packaging and transportation as well as the automobile and agricultural machine industry.

The seriousness of malfunctions due to vibrations is, of course, greatest in moving manned vehicles where human life is at stake, and the above stated relationship between the development of the vibration test technique and that of aircraft and space vehicles is quite understandable, indeed. With the advent of jet propulsion, however, a completely new and serious type of environment was introduced. It was no longer satisfactory to test critical units by means of simple sinusoidal vibration signals. The vibration excitation caused by the jet and the air turbulence experienced by very fast moving vehicles shows characteristics which are widely different from ordinary periodic motion. The excitation frequency spectra of these types of vibration environments are more or less continuous, and the instantaneous amplitude distributions can only be described with the aid of probability concepts. This called for a completely new test technique, *the technique of random vibration*. To-day random vibration test specifications are issued from various sources and the specifications are in general based on the use of an electrical wideband noise generator as signal source. However, the actual test technique has been subject to considerable changes in the years which have passed, and even if the art of random vibration testing is relatively young, it has reached a stage of surprising complexity.

The Vibration Machine.

Common for all types of present-day vibration testing is the use of an electrically controlled vibration machine which produces the mechanical motion to which the test object is subjected. The control of the machine will be detailed later on while this section will concentrate on some typical characteristics of the vibration machine itself.

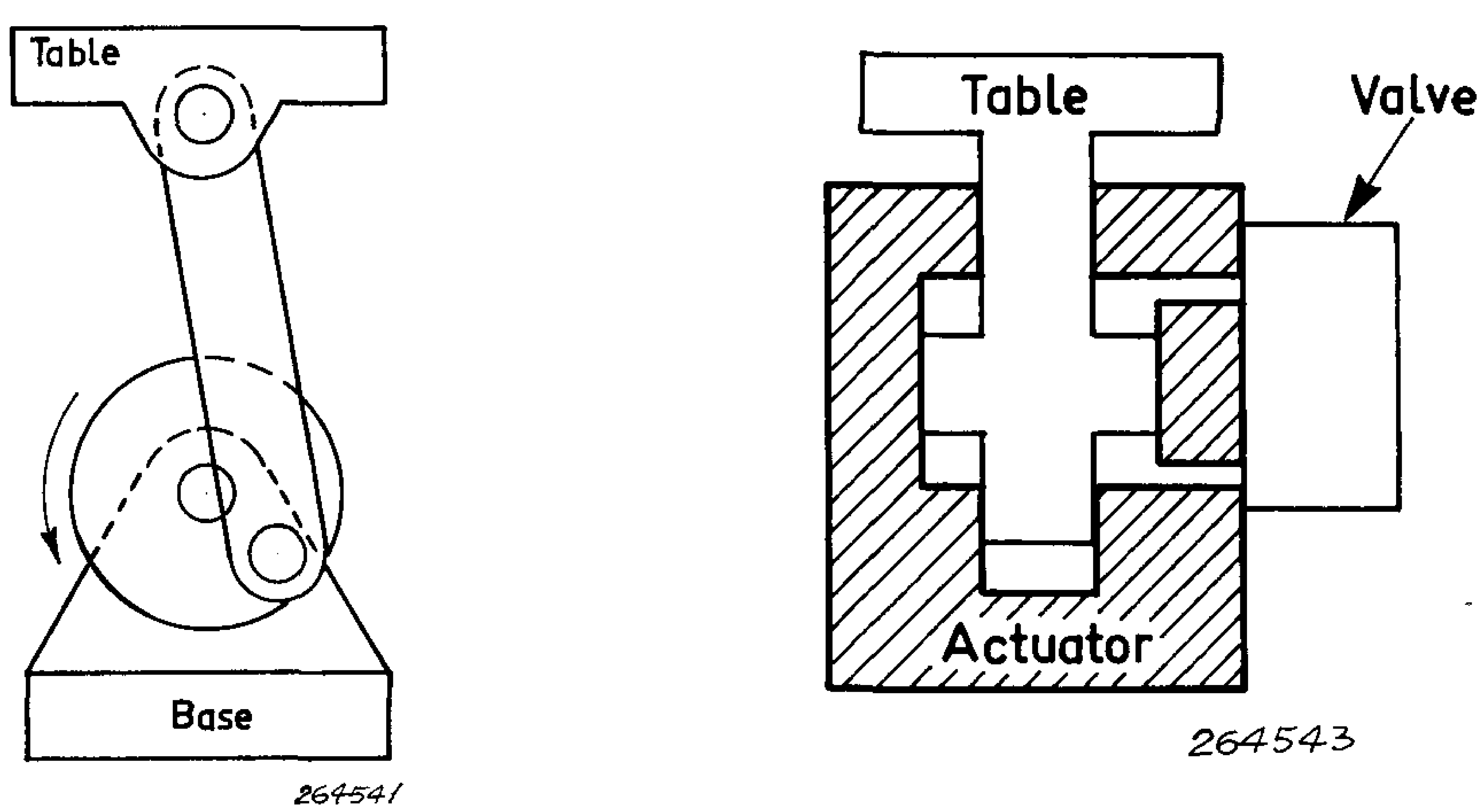


Fig. 1a. Example of a direct-drive vibration machine.
b) Principle of operation of an electro-hydraulic vibration machine.

One of the earliest types of vibration machines is the so-called *direct-drive machine*, Fig. 1a. It operates by varying the distance between the base and the load (test specimen) by means of an eccentric mechanism, and will develop almost any force necessary to move the load over this distance. The limitations of the machine are generally set by the strength of the driving mechanism (normally an electric motor) and by stalling of the drive. Two great disadvantages, however, seem to have more or less obsoleted this type of machine to-day, even though it can produce rather large amplitude variations: The machine can only produce periodic motion, and its useful frequency range is relatively narrow. Other types of direct-drive machines have been developed but are of very little importance in modern vibration testing.

The next type of vibration machine to consider is the *electrohydraulic vibration machine,* Fig. 1b. Here the load is attached to one end of a vibrating piston which is moved up and down within the cylinder by varying hydraulic oil pressure in two chambers. An electrohydraulic servo valve provides this varying pressure in conformance with an electrical control signal. This type of machine has certain advantages and is often used to-day. First of all it has a rather great displacement capability (strokes up to 30 cm), a very high force capability (over 50 tons of vibrational force!) combined with a reasonably wide frequency range, and it has practically no low-frequency limit (DC-coupled). Typical performance limitations of an electrohydraulic vibration test system are shown in Fig. 2. From the figure its main disadvantage can be immediately seen: the limitation in high frequency performance. Also, waveform distortion, hydraulic fluid purity and leakage in the oil system present problems.

The limitation in high frequency performance of the electro-hydraulic vibration machine has caused an intensive development of another type of vibration machine, *the electrodynamic vibrator.* Even though a high frequency limitation is imposed also upon this type of machine the limit is considerably higher than in the case of electro-hydraulic vibrators, see Fig. 3. In the figure a performance comparison is made between the types of vibration machines mentioned in the preceding text, and it is readily realized why the electrodynamic vibrator has become the most popular type of vibration machine in use at present. Except for some special applications of vibrations test

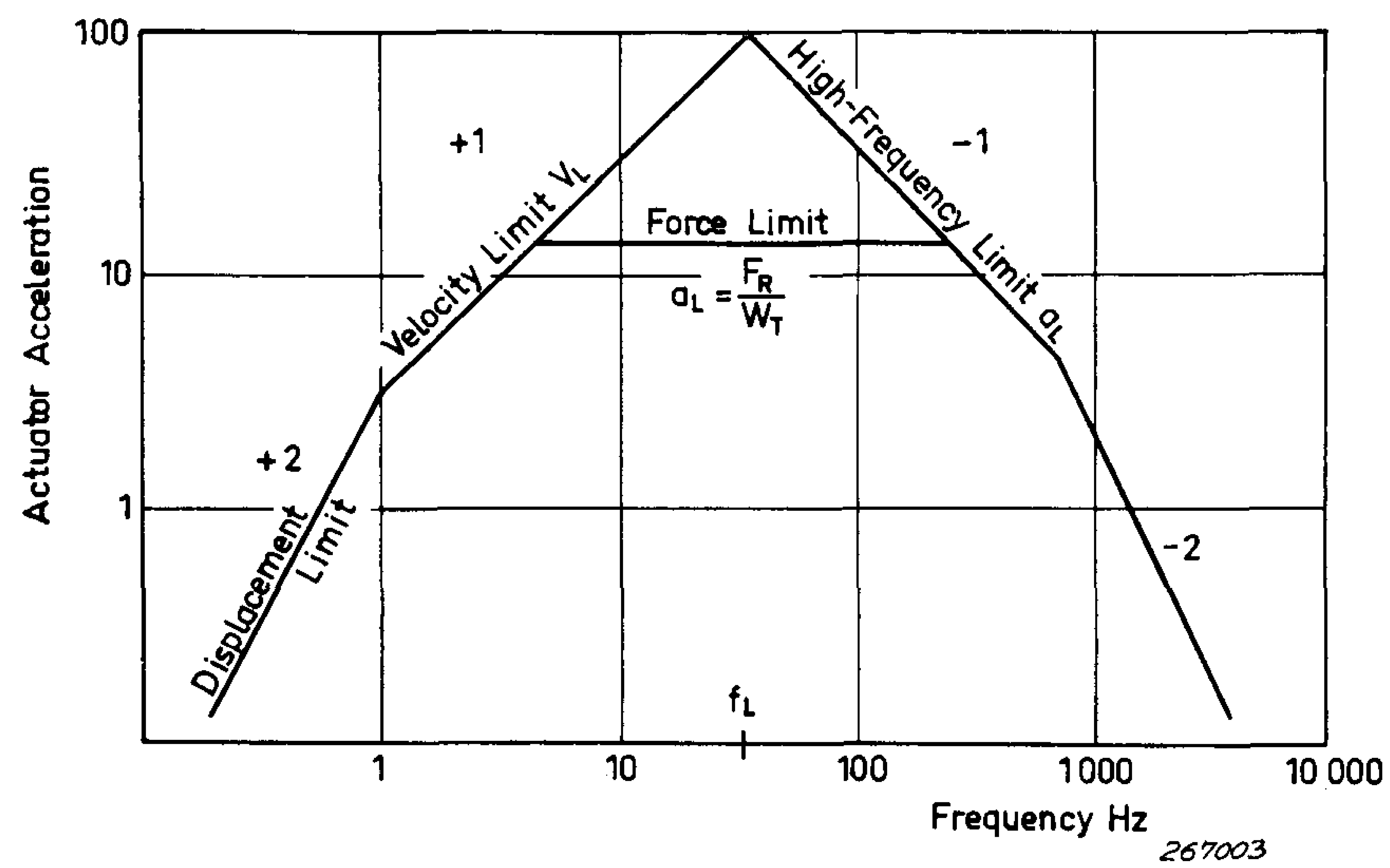


Fig. 2. Typical performance limitations of electro-hydraulic vibration test systems (G. B. Booth).

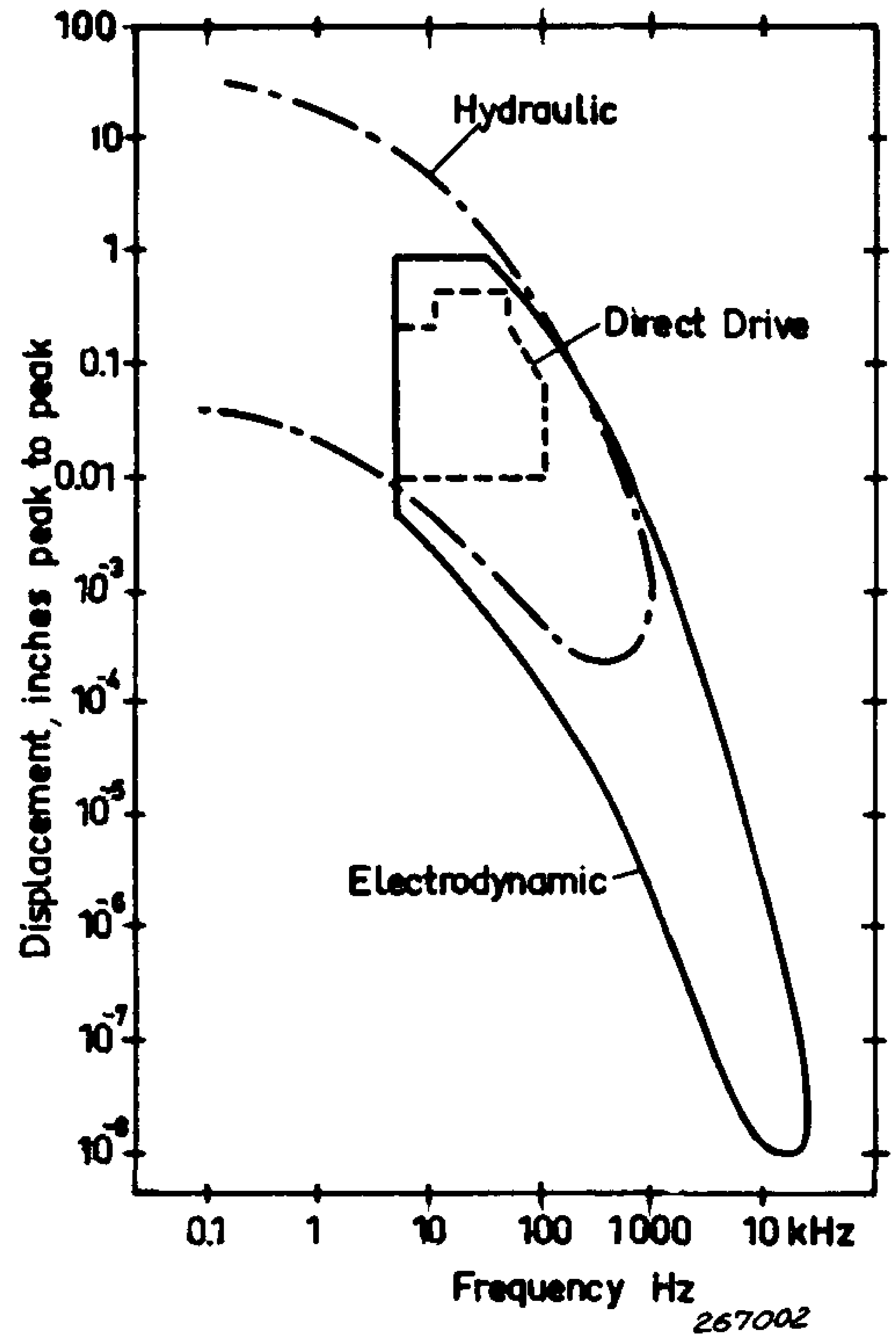


Fig. 3. Useful operating regions of available vibration machines (G. B. Booth).

techniques, where large displacement strokes at relatively low frequencies are required, such as for instance in the simulation of railroad boxcar motions and biomedical studies of seasickness in humans, the electrodynamic vibrator is by far the most widely used vibration machine. In the following some details concerning the operation of this type of vibrator will therefore be presented. Fig. 4 shows a sketch illustrating the basic design of an electrodynamic vibration machine. It consists of a magnet which produces the required constant magnetic field, a coil which is fed from an AC signal source, the moving element (on which the coil is mounted), and the flexures holding the coil and moving element in position, with respect to the constant magnetic

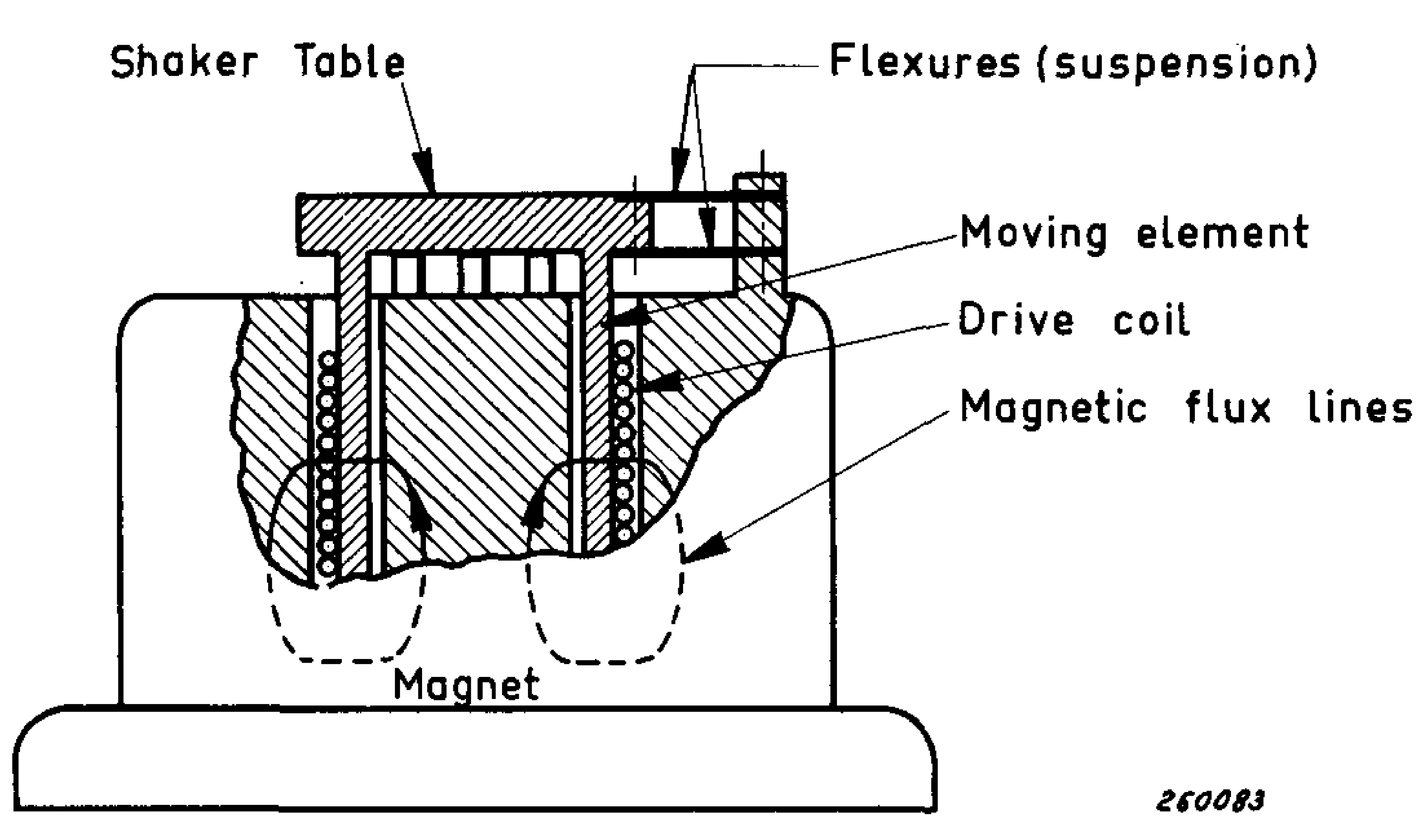


Fig. 4. Basic design of an electro-dynamic vibration machine.

field. The magnetic field strength and the coil diameter, number of turns and current determine the force available. This force is limited by the cooling provided for the coil and the materials and mechanical strength of the moving parts.

Ideally the coil and moving element should be a rigid unit where all points move in phase. Additionally, the ideal suspension of the moving element should be of such a nature that only a one-dimensional movement takes place, and finally the loading of the vibrator by a test specimen should in no way influence the motion. Unfortunately, it is impossible to fulfil these ideal conditions and compromises are therefore necessary.

As all bodies are more or less elastic, the requirement of rigidity must be compromised with the size and design of the moving element. This size is pre-determined by the size and shape of the specimen under test, as this also influences the movement of the moving element. To achieve pure one-dimensional translatory motion, the moving element must either be suspended in such a way that no other modes of movement is possible or forces which may cause such motion, must be eliminated.

The positioning of the test specimen on the table also greatly influences the mode of vibration. For example, if the dynamic center of gravity of the test specimen, at any particular test frequency, deviates from the axis of the moving element, rotational moments are produced, which will cause non-translatory motion of the specimen and moving element. Similarly, if parts of the specimen or moving element resonate at some particular frequency, within the test range, the center of gravity may "shift" during the test and non-translatory movements may be developed.

To minimize the influence of specimen resonances upon the motion of the moving element the "effective" mass of the moving element should be large. However, this means that to produce a certain acceleration of the specimen a rather high force is necessary. Furthermore adding mass to the moving element normally reduces the useful frequency range of the vibration machine. By introducing variable compensation networks or servocontrol of the moving element motion it is possible to "compensate" for the variation in load, with frequency caused by specimen resonances. Smaller moving masses can then be used for the same test.

If servocontrol is used rather than an increase in mass to minimize influence of resonances, two major advantages are gained:

1. A much greater part of the force produced by the vibration machine is transferred to the test specimen.
2. The useful frequency range of the vibrator can be extended because of the reduction in moving mass.

In Fig. 5 a typical frequency characteristic for a modern electrodynamic vibration machine is shown. The vibrator is driven from a constant current level source i.e. constant driving force is delivered to the moving element. The actual acceleration level of the shaker table is then plotted as a function

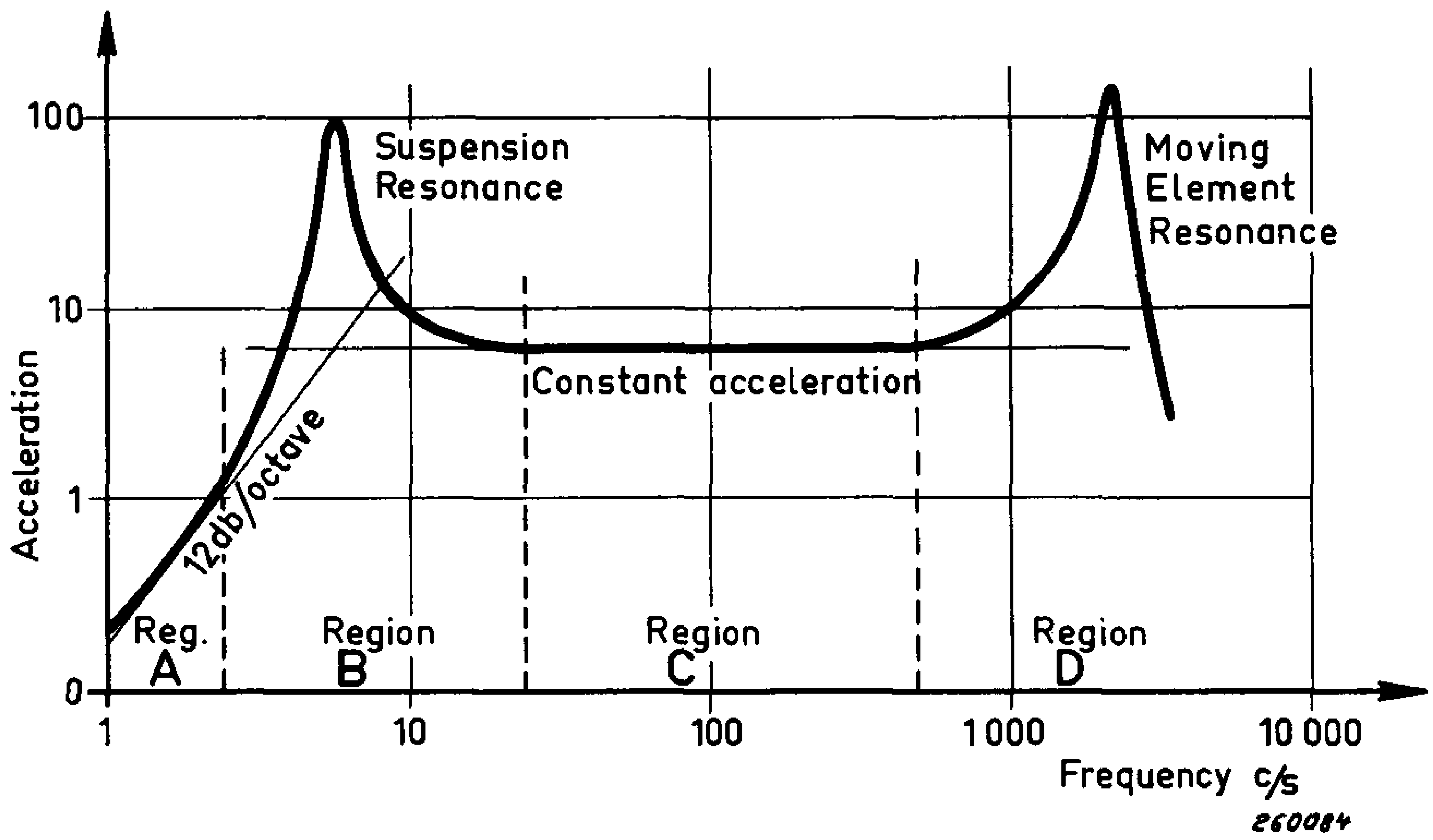


Fig. 5. Acceleration vs. frequency response of a vibration machine when the current in the drive coil is kept constant independent of frequency.

of frequency and the graph divided into different sections A, B, C and D which are described below.

When DC is supplied to the shaker drive coil, a constant force is developed which will cause the moving element to deflect, the magnitude of the deflection being determined by the stiffness of the mechanical suspension arrangement. At very low frequencies the deflection of the moving element will thus be stiffness controlled, i.e. constant displacement level conditions will be present at the shaker table region A in Fig. 5 (Constant displacement level conditions are represented by a slope of 12 dB/octave in the graph).

If the frequency of the drive signal is increased, the resonance of the overall mass of the moving element assembly and the suspension "spring" will cause a relatively great increase in the table's amplitude, region B in Fig. 5. Above the suspension resonance the mass of the moving element will control the table motion and a region of constant acceleration is developed, C, in Fig. 5.

At still higher frequencies the different parts of the moving element itself

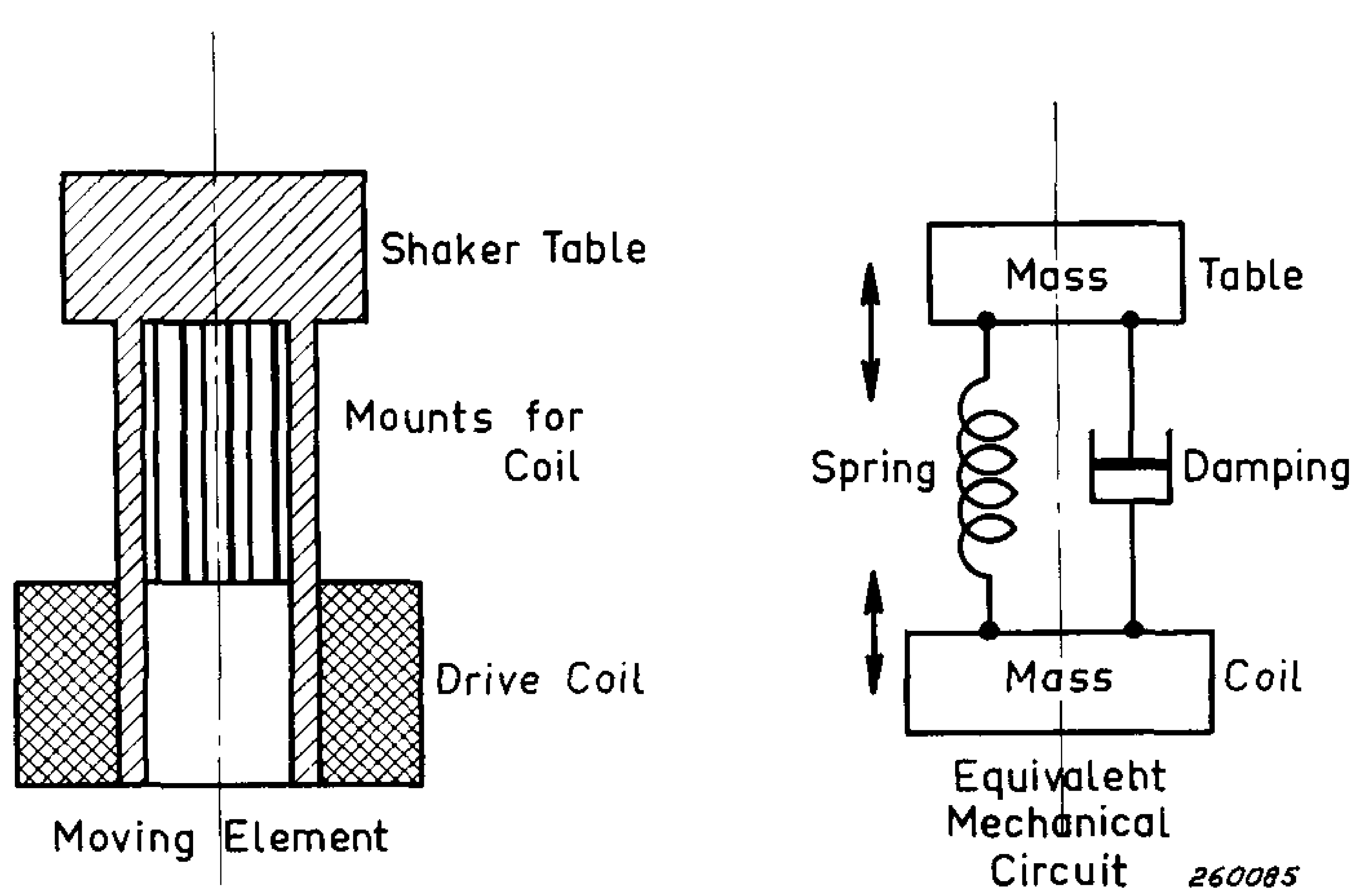


Fig. 6. Sketch illustrating how the basic axial resonance of the moving element of large sized vibration machines is produced.

will resonate and cause major irregularities in the frequency characteristic. In Fig. 5 only one moving element resonance is shown. The main resonance is normally of the axial type, and is produced by the spring/mass system of the moving element in the axial direction, see also Fig. 6. This resonance limits the upper end of the useful frequency range.

The acceleration vs frequency characteristic of electrodynamic vibrators is more or less similar for all vibration machines when the machine is driven from a constant current level source. However, amplifiers which effectively transform the available electrical power into mechanical force may usually be well approximated by a constant voltage level source. The acceleration vs frequency characteristic of electrodynamic vibration machines subjected to these conditions varies considerably, depending upon the electrical resistance of the drive coil winding and the mechanical damping of the suspension

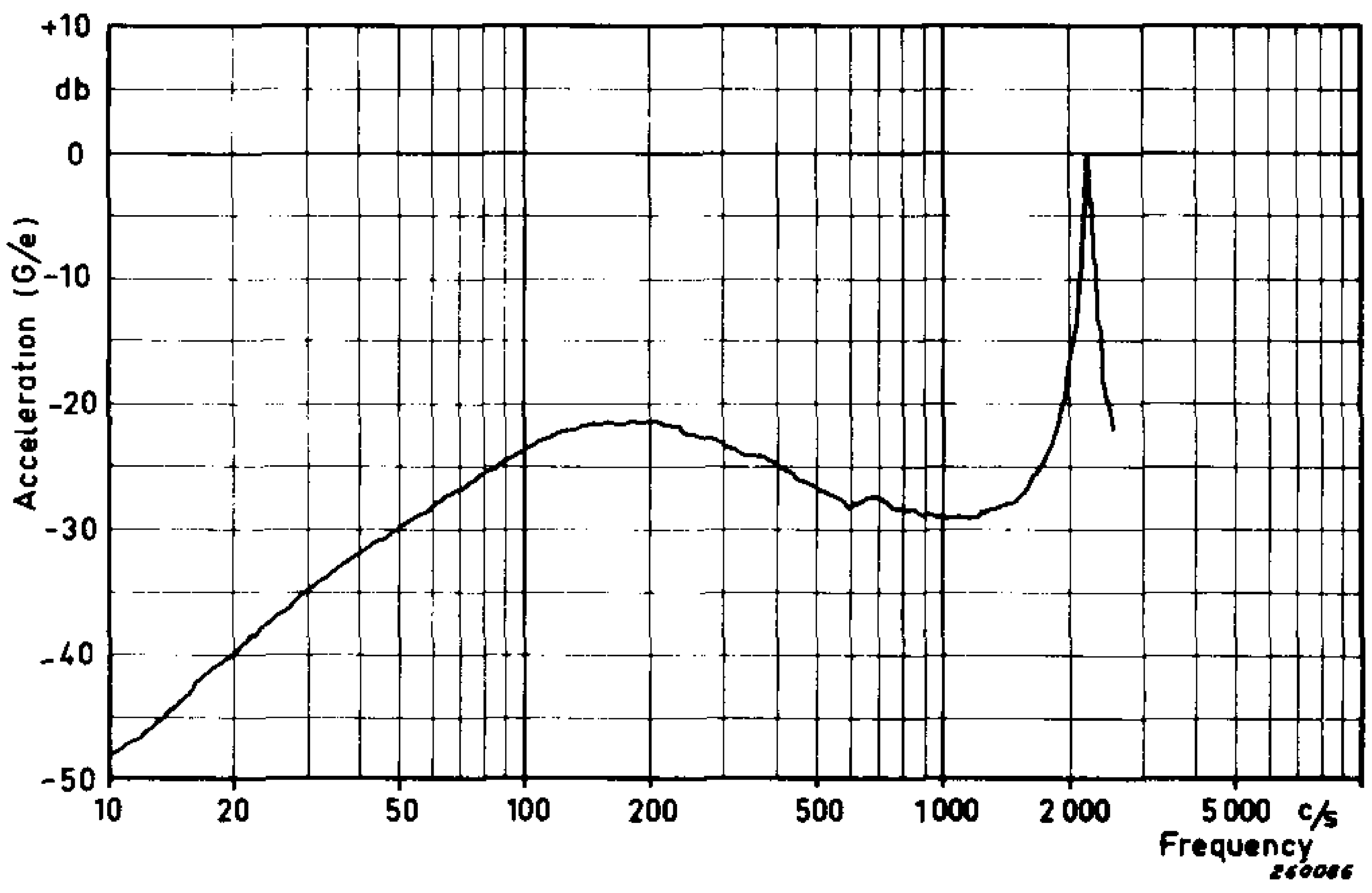
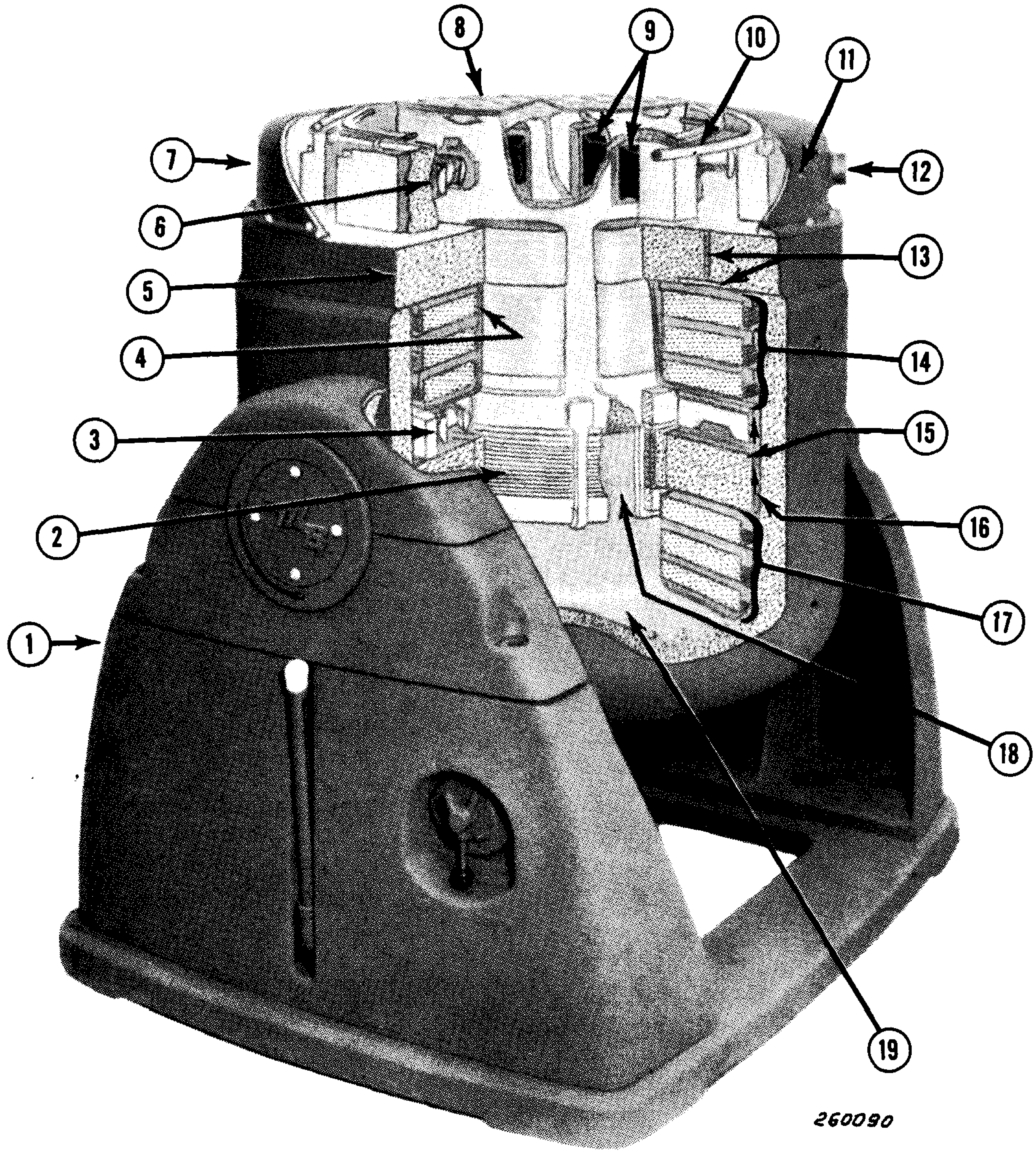


Fig. 7. Typical frequency response curve of a low-resistance, heavy-duty vibration machine with constant v o l t a g e level drive.

arrangement. In Fig. 7 the frequency response of a low resistance well damped vibration machine is shown, the drive coil of which was fed from a constant voltage level source.

In the case of the low-resistance, heavy-duty vibration machine the suspension resonance is completely eliminated due to the electrical damping effect. This damping effect is caused by the low output resistance of the power amplifier which almost short-circuits the back e. m. f. induced in the coil, when it moves in the constant magnetic field of the shaker. As the back e. m. f. is proportional to the velocity ($e \sim \frac{d\phi}{dt}$) the movement of the moving element in this frequency region (corresponding to region B in Fig. 5) will be velocity controlled, and the acceleration vs. frequency characteristic shows a slope of 6 dB/octave. At higher frequencies, where the movement of the moving element is mass-controlled, the acceleration level of the element will be constant. However, as the electrical resistance of the coil winding is low, the constant acceleration level region is normally small, and the drive coil inductance causes the



1. Pedestal
2. Driver Coil
3. Lower Rockers
4. Fiber Glass Coil Enclosure
5. Top Cover Casting
6. Upper Rockers
7. Dome
8. Table & Moving Element Ass'y
9. Rubber Flexures
10. Ring Cooling Assembly
11. Accelerometer Connector
12. Power Receptacle
13. Clamping Screw, Pressure Pad
14. Upper Group of Field Coils
15. Cover Assembly
16. Oil Flow Channel
17. Lower Group of Field Coils
18. Pole Piece
19. Body Casting and Cavity

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Fig. 8. Cut-away view o a heavy-duty vibration machine.

acceleration vs. frequency characteristic to drop off with frequency. The "middle" region of the response curve shown in Fig. 7 will therefore show the characteristics of a very well-damped resonance. As explained above, this resonance is caused by a combination of mechanical and electrical factors and may be termed "electro-mechanical resonance". (By some manufacturers the phenomenon is designated as "electrical resonance").

At high frequencies the importance of the drive coil reactance increases ($X_c = 2\pi fL$) and the acceleration vs. frequency characteristic drops off until the resonances of the moving element start to influence the table motion. As examples of present day vibration machines Fig. 8 shows a cut-away view of a modern, heavyduty vibrator, while Fig. 9 illustrates an application of one of the larger vibration machines built to date.

Vibration Test Procedures. The Sweeping Sine Wave Test.

As mentioned in the introduction the techniques used in the field of vibration testing have changed considerably over the past decades, from pure sinusoidal testing at fixed frequencies to complex random vibration test schemes. However, one of the "older" tests that is still very popular, and which is likely to remain so, is the sweeping sine wave test. The main reasons for the popularity of this test procedure are that it is a highly efficient tool in search of dangerous resonances, especially in conjunction with a stroboscope, and that the electronic control equipment used for the test is relatively inexpensive, although it may become quite involved.

Before going into a more detailed description of some of the methods used to produce and control frequency sweep tests the basic block diagram shown in Fig. 10 should be considered. The arrangement shown contains the essential ingredients of a frequency sweep test system. It consists of an electronic generator (oscillator) the frequency of which is continuously changed during testing, a power amplifier and the vibration machine itself.

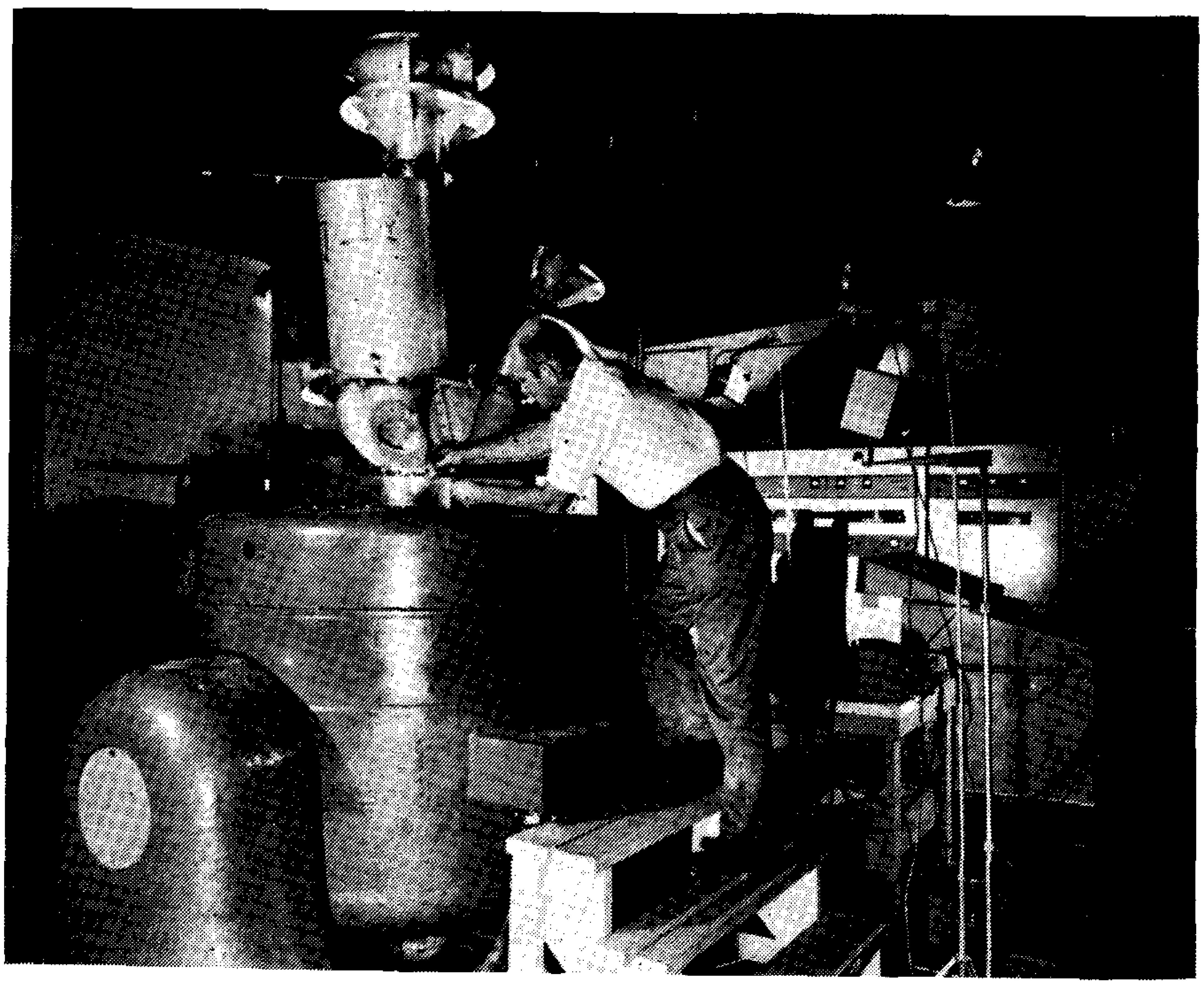

Fig. 9. Mounting of the test specimen on a large vibration machine.

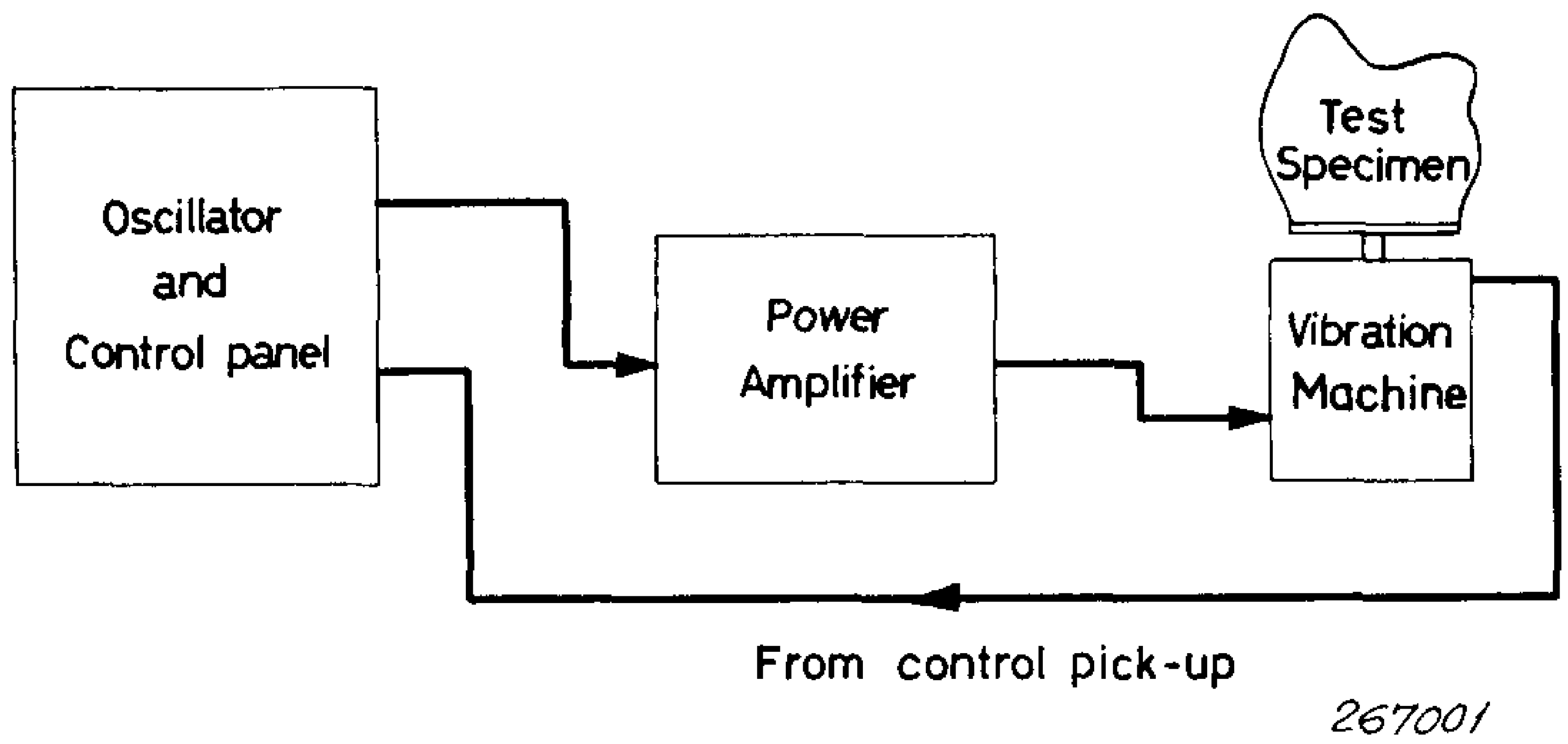

Fig. 10. Block diagram of a typical frequency sweep test arrangement.

Due to resonances in the test specimen, Fig. 11, and the vibration machine, the power necessary to subject the test specimen to a certain, constant vibration level will not, however, remain constant during the test, but will be a function of the frequency of the vibrations. This is further illustrated in Fig. 12 which shows a plot of the acceleration of the shaker table as a function of frequency for constant voltage drive of the vibration machine and a table load consisting of a single degree of freedom mechanical system. To keep the vibration level constant the output from a vibration pick-up mounted on the table is used to control the input power to the vibrator, see also Fig. 10.
Normally, the control of the vibration level is made in such a way that when the vibration level of the table tends to increase, which would cause the output voltage from the control pick-up to increase, the input power to the vibration machine is automatically decreased until the same vibration level is regained as was present before the change in vibration level occurred.
The automatic decrease in exciter input power will not follow instaneously when an increase in vibration level is felt by the control pick-up, but it will take a certain amount of time to regain the original level. This time constant of the regulation, or in other words, "the regulation speed" should be selected according to the expected Q-values of the system resonances and the sweep rate chosen for the frequency sweep i.e. the regulation speed must be greater than the speed with which the system resonances are built up.
In practice this means that the highest regulation speed should be used, the

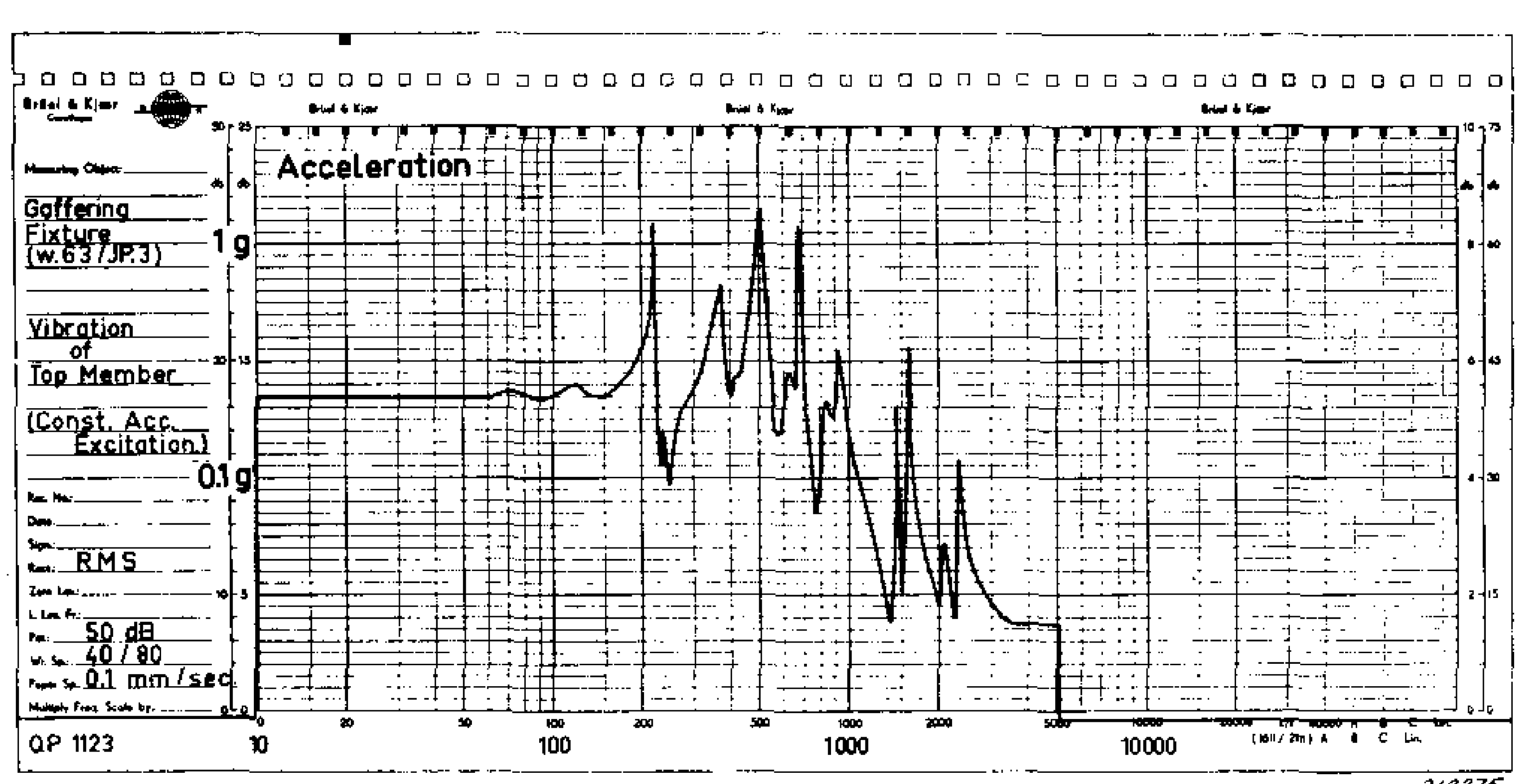

Fig. 11. Example of the frequency response of a complicated test structure.

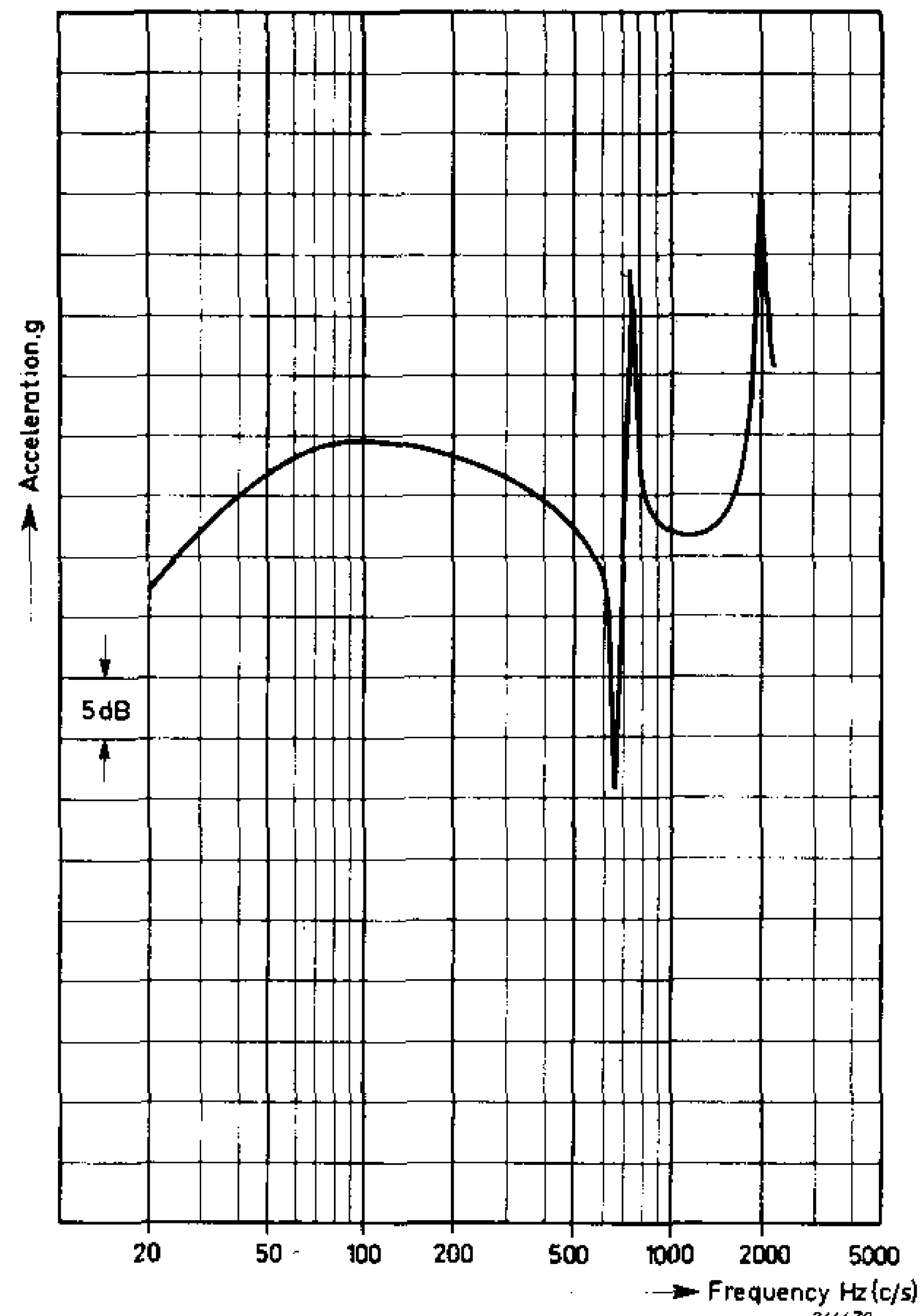

Fig. 12. Typical acceleration level vs. frequency curve measured on the shaker table of a vibration machine loaded by a single degree-of-freedom system.

upper limit of the regulation speed being set by interaction between the regulation speed and the actual vibration frequency (causing distortion).

To allow for optimum automatic regulation the regulation speed must be variable and preferably provision should be made for programming the regulation as a function of the frequency sweep. A practical arrangement, including an electronic generator which allows such programming to be made, is shown in Fig. 13.

The servo system illustrated in Figs. 10 and 13 enables not only a regulation of the vibrator frequency response with regard to resonance effects but it is also possible to include in the servo loop certain frequency dependent networks so that various predetermined test programmes can be carried out under controlled conditions.

Either an accelerometer or a velocity pick-up may be used as transducer and

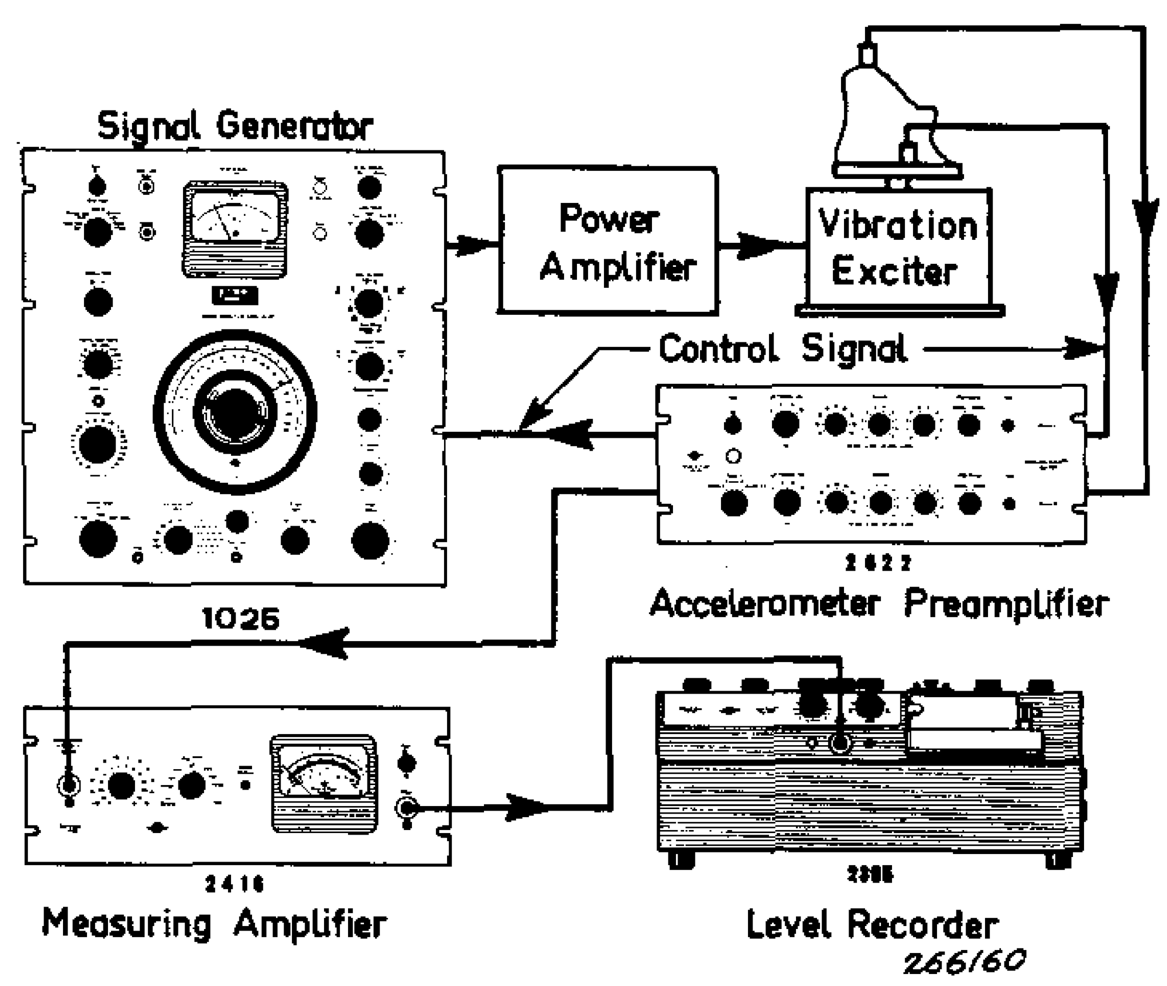

Fig. 13. Vibration test system in which the vibrations of the specimen are plotted on a Level Recorder.

the vibration derivation section of the oscillator shown in Fig. 13 contains all the integrator and differentiator networks necessary for deriving acceleration, velocity or displacement from either input. It is possible to hold any one of these dynamic properties constant on the shaker table, and also there are two "auto" arrangements in which the quantity measured and controlled can be changed at a predetermined "cross-over" frequency, i.e.

displacement below cross-over to acceleration above,
or velocity below cross-over to acceleration above.

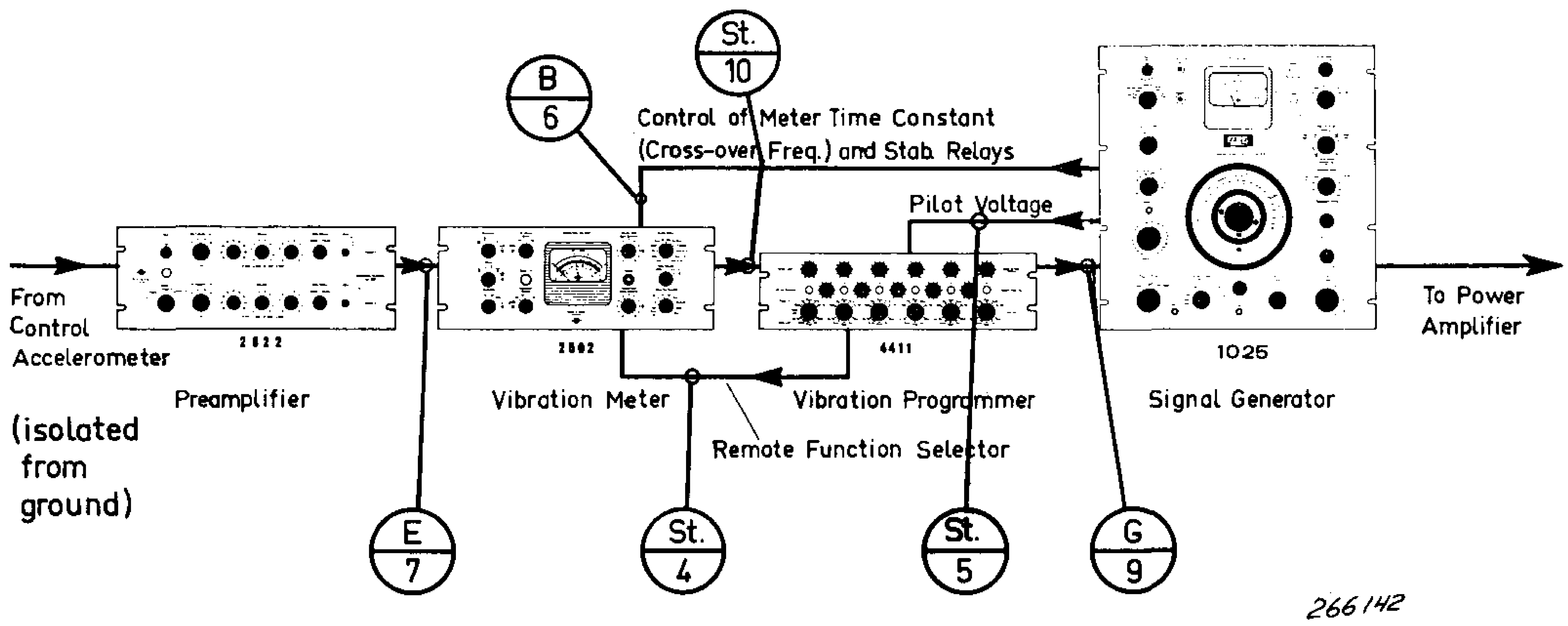

Fig. 14. Vibration test system allowing elaborate programming of test levels with frequency.

As the test specifications become more exacting with regard to excitation versus frequency characteristic it is, however, often found desirable to have more cross-over frequencies than the one mentioned above. Also, it might be necessary to control the excitation over at least parts of the full frequency sweep of the generator by specially designed networks. An arrangement which enables the inclusion of such networks and, furthermore, allows the use of up to five adjustable cross-over points is shown in Fig. 14. As can be seen from the figure it is then not only necessary to include a special "Vibration Programmer" in the test set-up but use must also be made of an external vibration meter which accomodates the five cross-over points.

To illustrate a possible sweep program utilizing more than one cross-over point the curve given in Fig. 15 should be considered. Test curves like the one shown are developed from environmental studies and will be discussed further in conjunction with the derivation of appropirate vibration test specifications for frequency sweep tests.

When large, complex test specimens are bolted to the vibration machine one finds that feedback control with a single vibration control pick-up, as has been discussed up to this point, is not sufficient. The specimen is not a dead mass, and different parts of the structure resonate at different frequencies, often causing an irregular overall motion. If a single vibration pick-up controls the motion of the shaker table, portions of the test specimen may be severely overtested, while other portions are undertested. Normally the motion of the

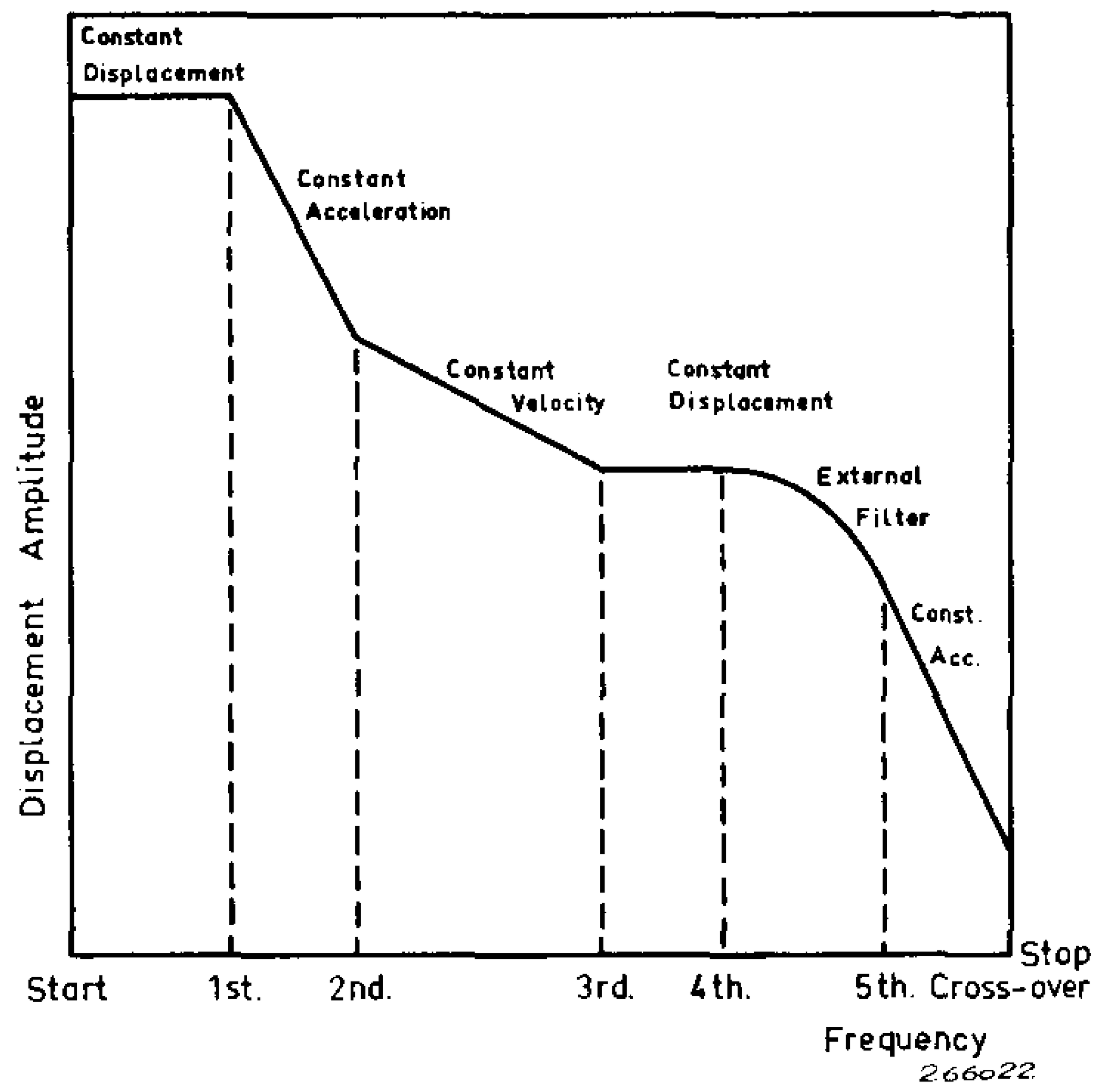

Fig. 15. Example of a possible frequency sweep/test level programme.

most important fixing points of the specimen should be controlled in one of the ways outlined below.

1. To reduce overtesting an overall control signal can be used which is an average of the vibration signals from the fixing points. If the averaging is done on an RMS basis this reduces the over-testing to a factor less than n, where n is the number of input points. Using for example four accelerometers at four input points will reduce overtesting to less than 12 dB. Undertesting will only be slightly affected.
2. If a further reduction of overtesting is found necessary, this can be done by switching circuits automatically selecting the largest of the signals from the measuring points. Overtesting is thereby reduced to zero. Undertesting is not remedied.
3. Undertesting due to different motion at input points can similarly be eliminated by selecting the smallest of the vibration signals for feedback. This procedure does not necessarily affect overtesting.

Thus, the availability of equipment which has facilities for feedback control via the average, the highest or the lowest of at least four accelerometer signals is highly desirable. Such an instrument has been developed by Brüel & Kjær in the form of the "Control Signal Selector Type 4410", and use of this instrument is illustrated in Fig. 16.

Finally, Fig. 17 shows a test arrangement utilizing both a "Vibration Programmer" and a "Control Signal Selector". As can be seen the electronic part of the test arrangement is now beginning to become rather complex! Of course, other combinations of the control principles described in the preceding text may also be used, and when complicated test schemes are carried through on complex test specimens the setting-up and maintenance of the test equipment requires a certain skill and knowledge of the test engineer in charge of the programme. Once, set up, however, the normal use of the equipment is relatively simple and easy to perform.

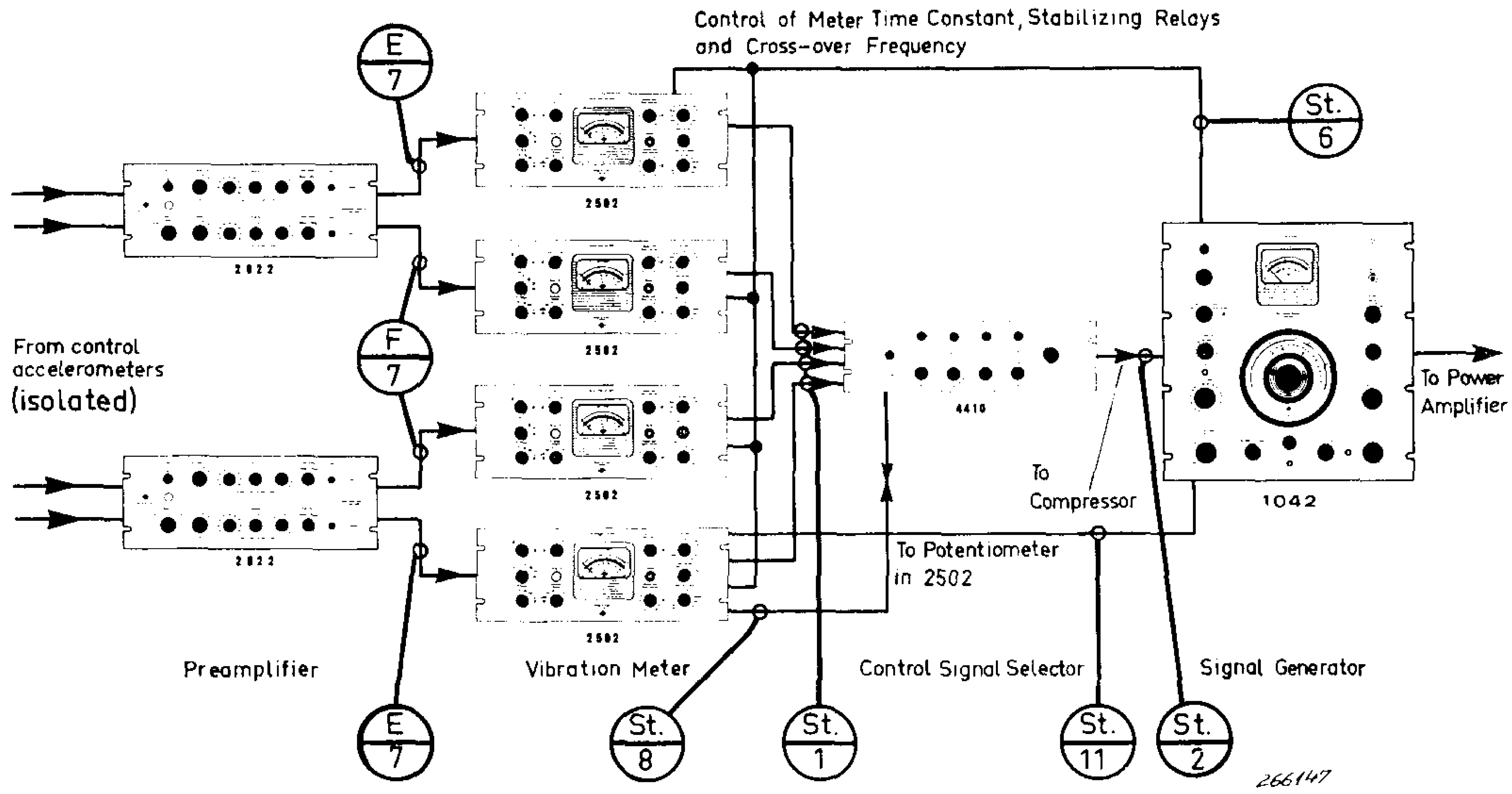

Fig. 16. Example of a vibration test system allowing test level control from more than a single point.

It was stated earlier in this paper that the development of vibration testing technique has followed, very closely, the development of aircraft and space vehicles. As the specimens to be tested grew larger and larger so also did the vibration machine, the largest electro-dynamic machine available to-day being capable of producing some fifteen tons of vibrational force. Even though it would be desirable in some cases to have even larger machines available it has not been found technically nor economically realistic to produce electro-dynamic vibration machines with larger force ratings than the above mentioned fifteen tons machine. One of the reasons for this is that as the test specimen

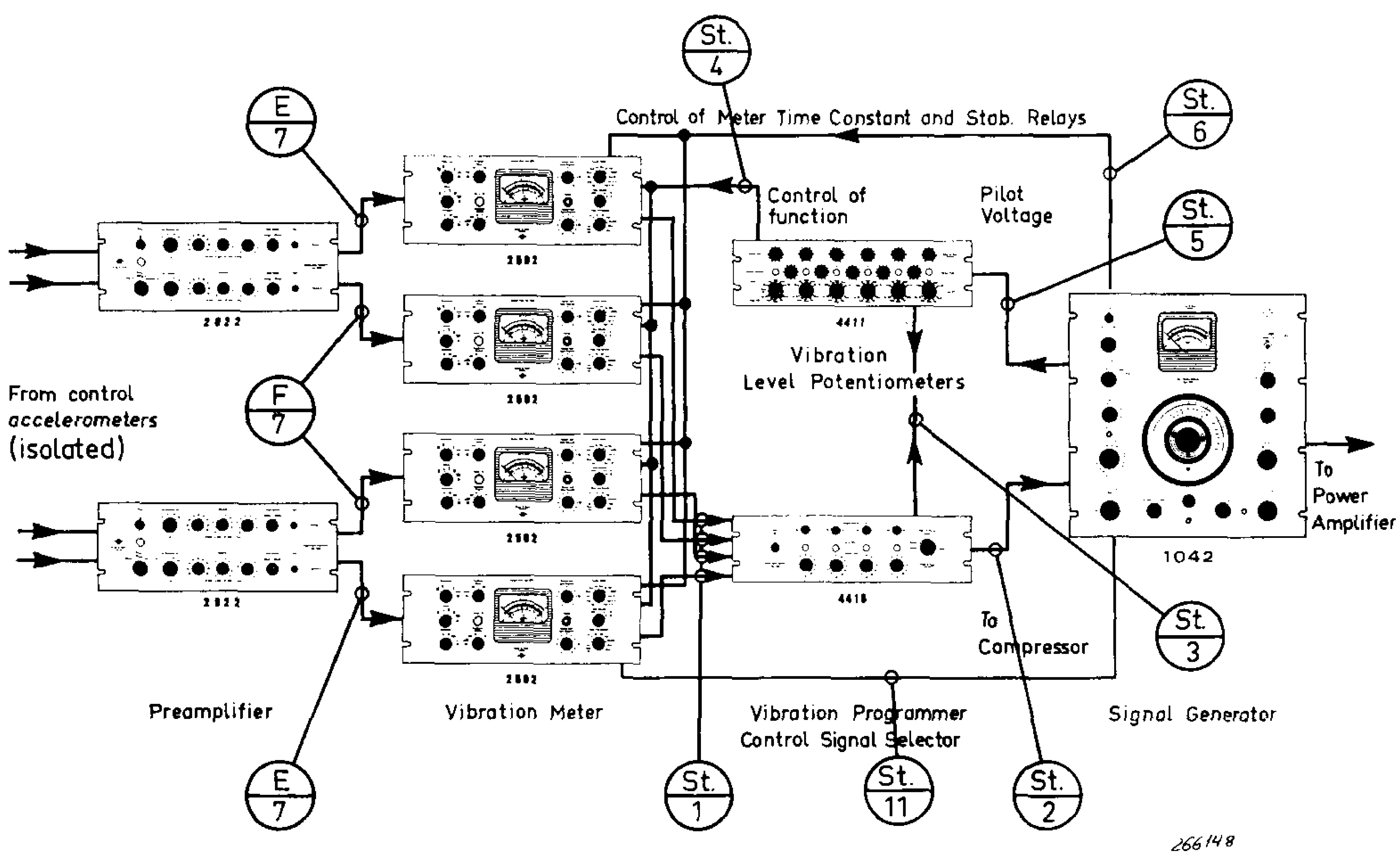

Fig. 17. Vibration test system allowing both complex programming of test levels and multipoint test level control.

becomes larger problems in mounting the specimen on the shaker table also become greater. The problem of testing large scale specimens is therefore presently being solved by the use of multiple vibration machines operated either in series or in so-called master-slave arrangements rather than by trying to build larger vibrators. Fig. 18 shows the principle involved in the control of a master-slave arrangement. It is seen that while the frequency

Principle of Setting Up **Principle of Operation**

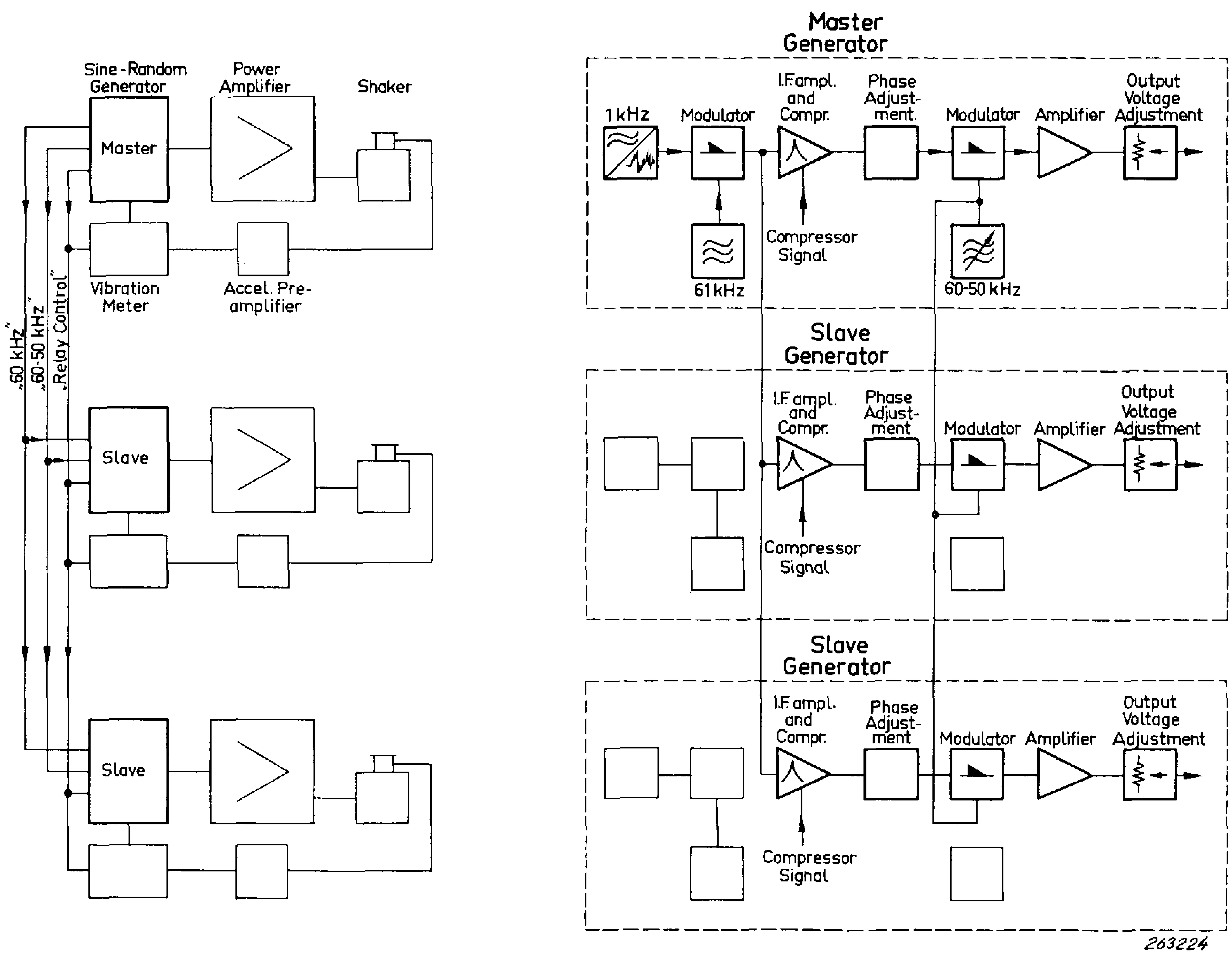

Fig. 18. Example of a "master-slave" system utilizing three vibration machines to excite the test specimen.

sweep is governed by the "master" all the "slaves" have separate control signal loops. Also in this case, of course, it is possible to apply the Vibration Programmer and Control Signal Selector technique to each vibration machine separately.

However, in many cases the test specimen might only have one fixing point to each vibration machine which makes the use of a Control Signal Selector superfluous. Large test specimens can in this way be tested to high vibration levels. A special feature of the arrangement shown in Fig. 18 is that the phase relationship between the "master" and "slave" vibration can be adjusted over a full 360° angle, thus producing various kinds of specimen mode excitation.

Before finishing this discussion on the sweeping sine wave vibration test, mention should be made of common methods used in the derivation of appropriate test specifications. Generally speaking vibration test specifications should be derived directly from the environment in which the part or assembly is supposed to operate in practice. This has, however, often proved to be a very difficult task partly because the environment is not always known from beforehand and partly because of insufficient and inaccurate

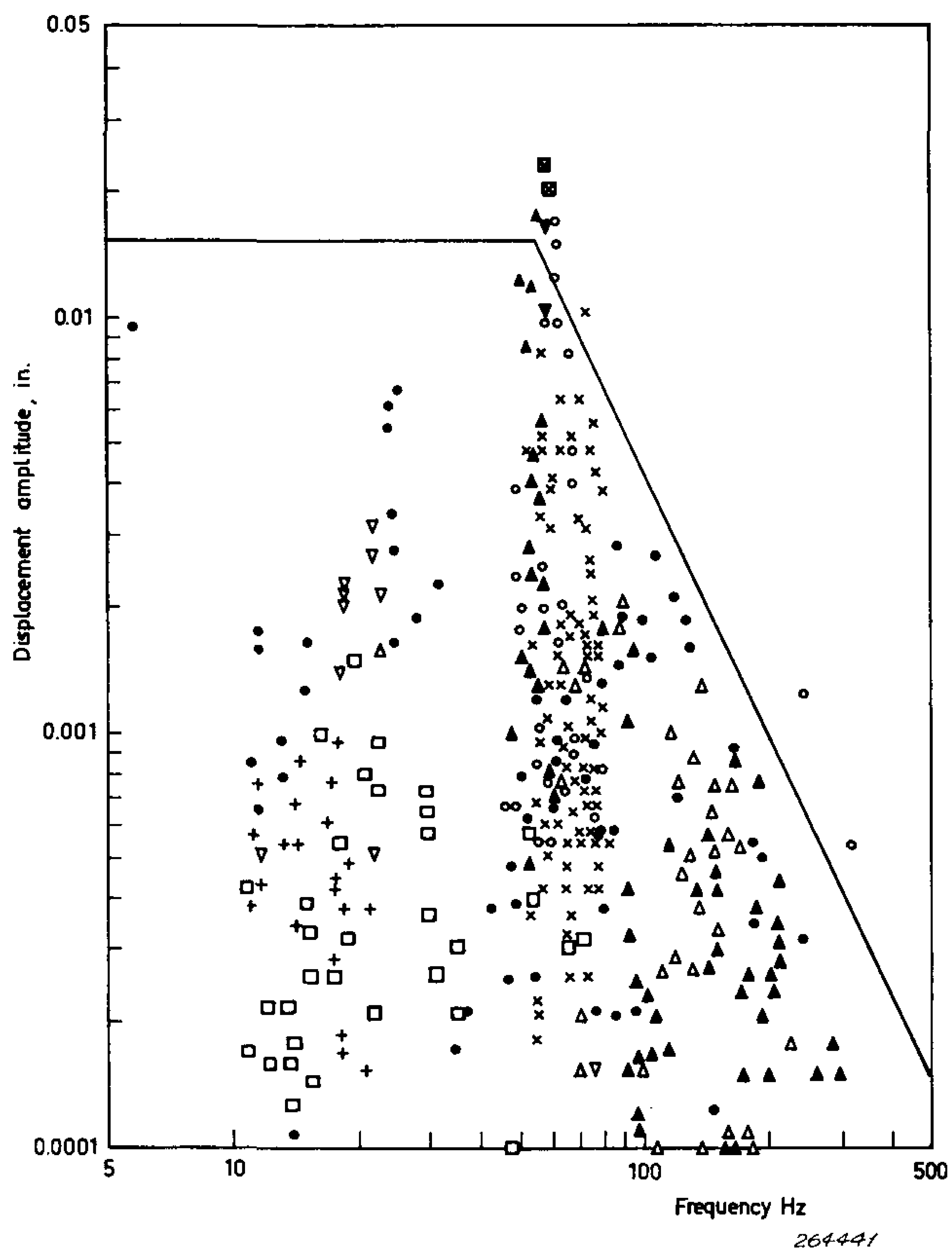

Fig. 19. Example of the derivation of test level specifications from environmental data. One "cross-over" point only is used in the test programme.

measurement data relating to the particular environment. Also the effects of mechanical impedances and the statistical nature of many vibration environments are often desregarded in the derivation of test specifications. The lack of exact environmental data has made it a common practice to collect all the measurement results relating to the environment in question and from the collection of data then draw some "estimated envelope" curves as illustrated in Figs. 19 and 20. As can be seen from the figures, some measurement points fall outside the "estimated envelope", a fact which is normally referred to as "a best engineering judgement".

Even though the method of "enveloping" seems to be a fairly sound method in developing test specifications, because of the many unknown factors involved in defining a vibration environment, it leaves a lot to be desired with

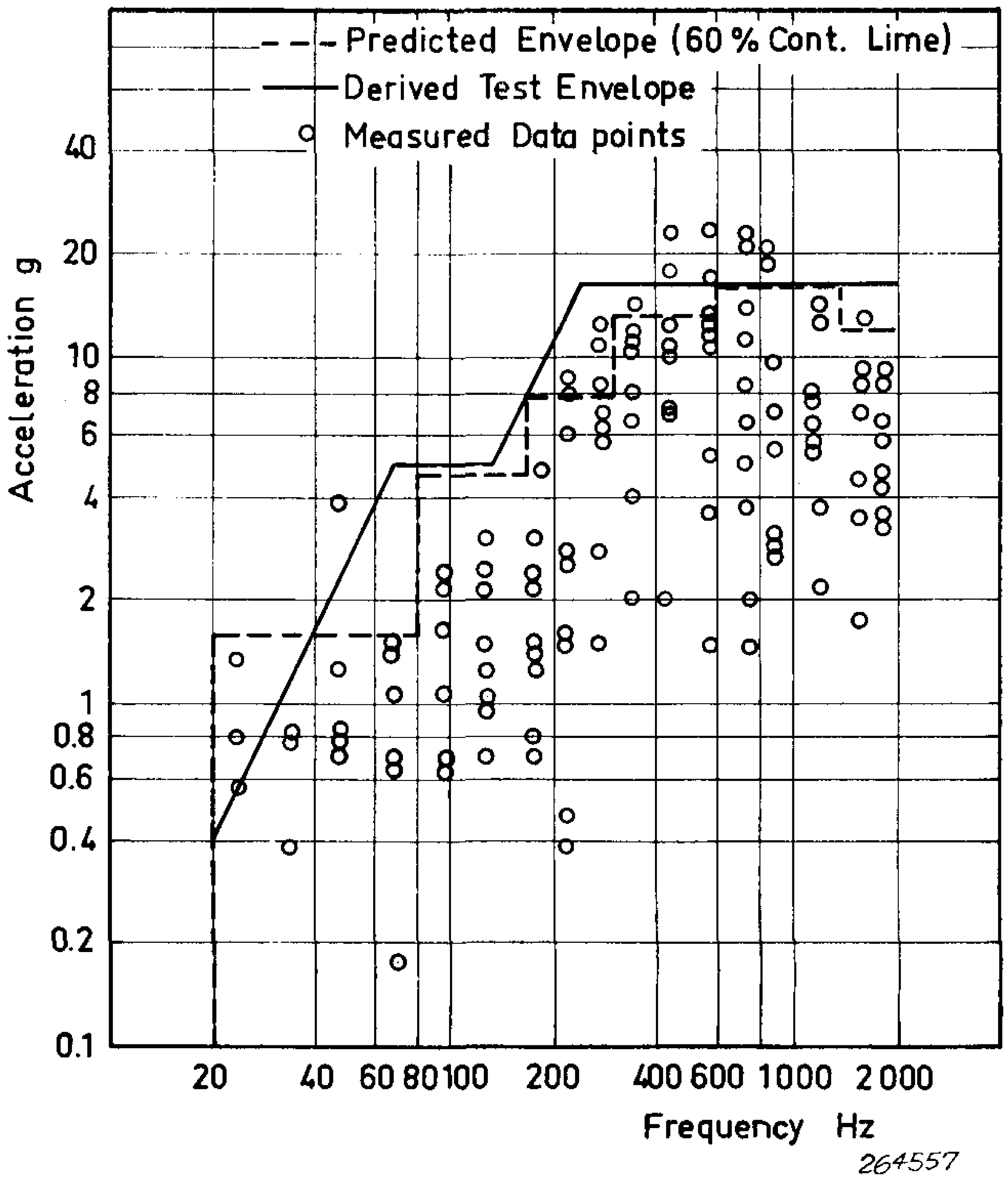

Fig. 20. Similar to Fig. 19 but exemplifying the use of more than one "cross-over" point in the sweep programme.

respect to precision. It is hoped, however, that future research with respect to mechanical impedance, vibrational power transfer and structural system responses will allow a *better* "engineering judgement" to be made than is possible to-day.

When a suitable test spectrum envelope is defined, see also Fig. 21, the question of the test duration remains. This greatly depends upon the purpose of the test and varies from some 10^7 cycles of vibration at a particular frequency (pure fatigue testing) to certain amounts of time spent in various

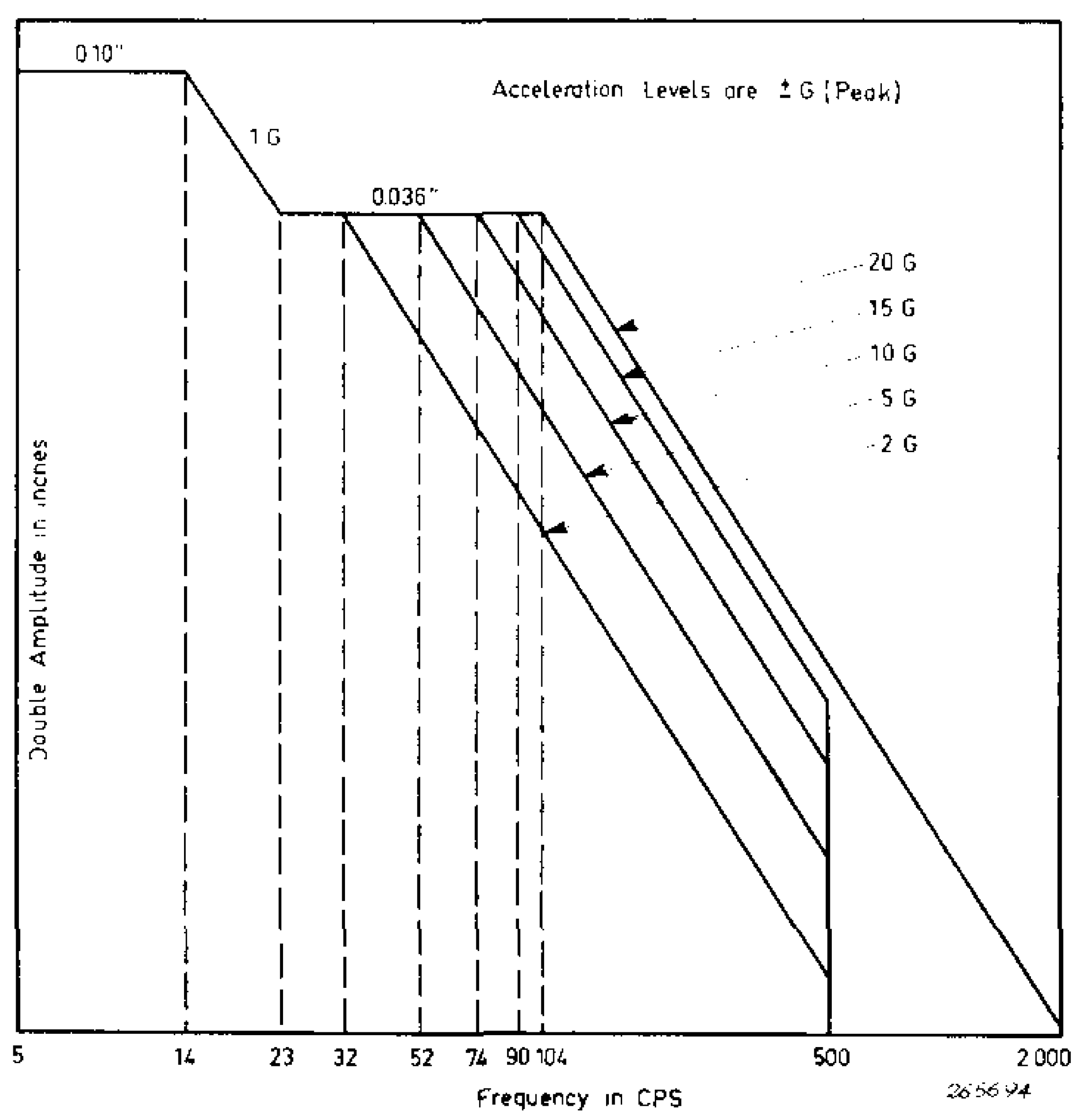

Fig. 21. Example of typical sweep test specification requirements.

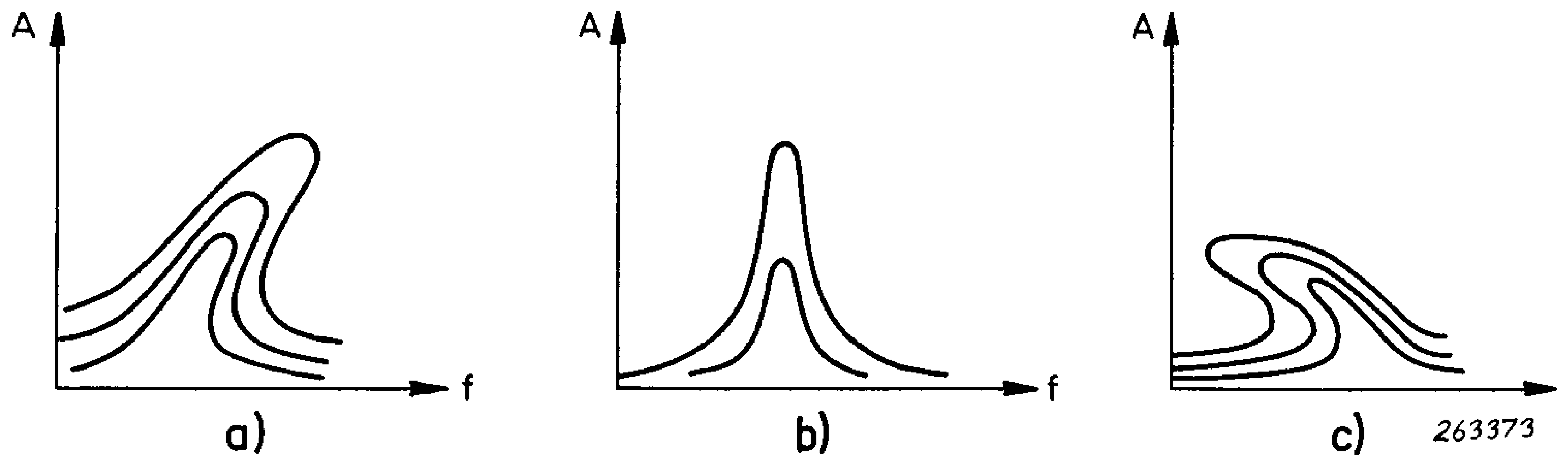

Fig. 22. Typical resonance curves for various levels of excitation for:
a) A hardening spring type resonant system.
b) A linear resonant system.
c) A softening spring type resonant system.

frequency regions. The latter requirement is then often "transformed" into a statement of one or more frequency sweeps at a certain sweep speed, normally defined in Octaves/Minute. The required amount of time spent in certain frequency regions may be developed on the basis of fatigue predictions, or simply from the fact that it is the expected period of time that the test specimen will have to withstand the vibration environment during actual operation.

In the sweeping sine wave test specification it is generally recommended to perform at least one upward and one downward frequency sweep. This requirement is related to the excitation of nonlinear specimen resonances, and is rather important because the excitation of such resonances depends greatly upon the direction of the sweep, see Figs. 22, 23 and 24. Figs. 23 and 24 refer to a very common type of nonlinear resonance, the socalled hardening spring type, and the response curves shown in Fig. 24 have been obtained in model studies performed at Brüel & Kjær.

The maximum sweep speed to be used also depends upon the Q-values to be expected in the specimen resonances. If the test is specified only in terms of time, it is generally adviseable to perform one upwards and one downwards sweep only, at the lowest sweep speed which accomodates the

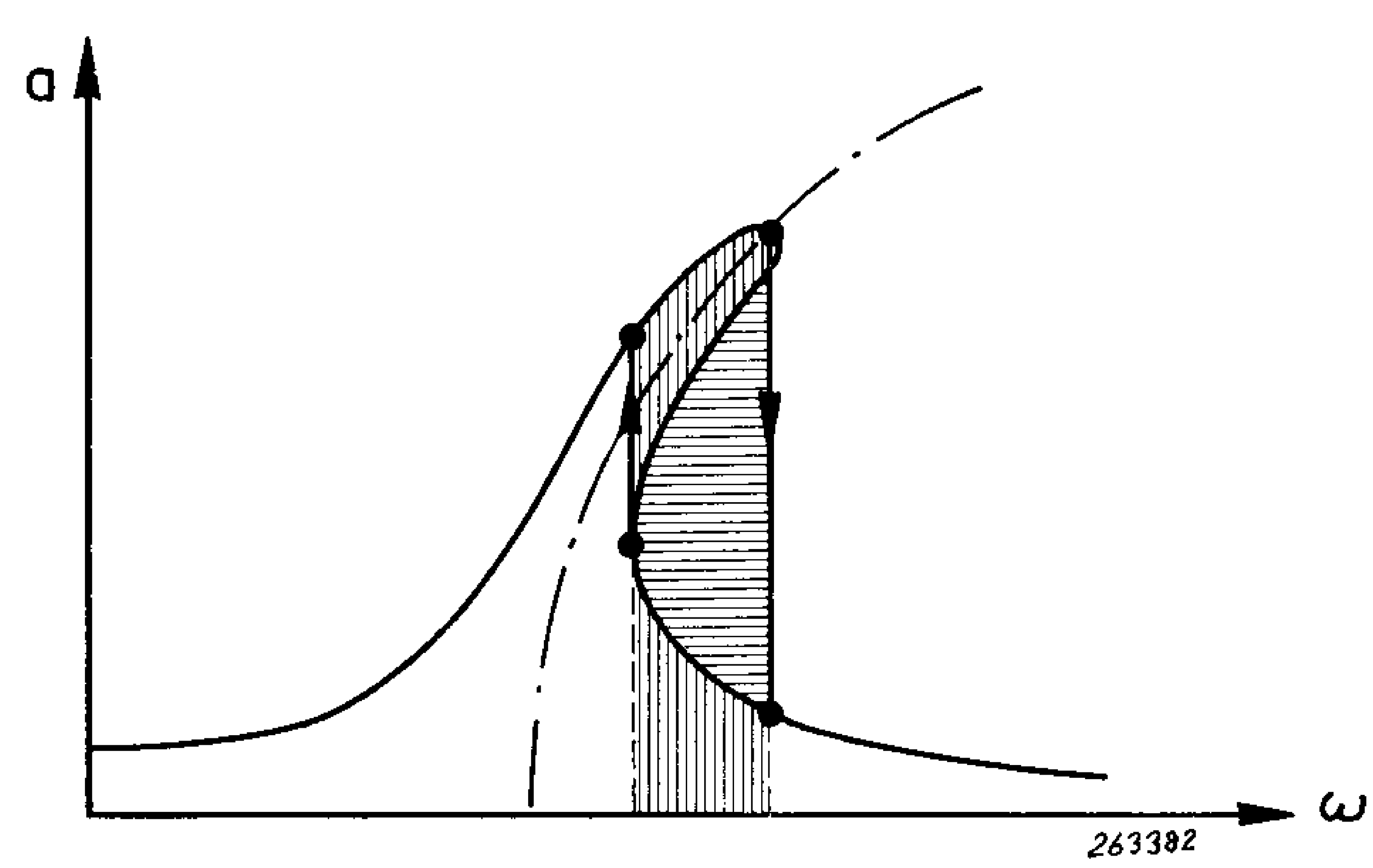

Fig. 23. Theoretical frequency response curve for a hardening spring type resonant system.

test time requirements. It is readily seen that the simple sweep test, which can only test a specimen sequentially with respect to frequency must necessarily take longer time than a test which excites the specimen simultaneously at all frequencies. Except for testing at the developmental stage, where the detection of dangerous resonances and the determination of their Q-values are of prime interest, the simple sweeping sine-wave test will often be found too time consuming. For this and other reasons the so-called wideband random vibration test has become increasingly popular as a qualification test during the later years. Also, a further type of testing, the multiple sweep random test is gaining interest. This test combines features from the wideband random and the sweeping sinewave test, lying somewhere in between the two in complexity and cost.

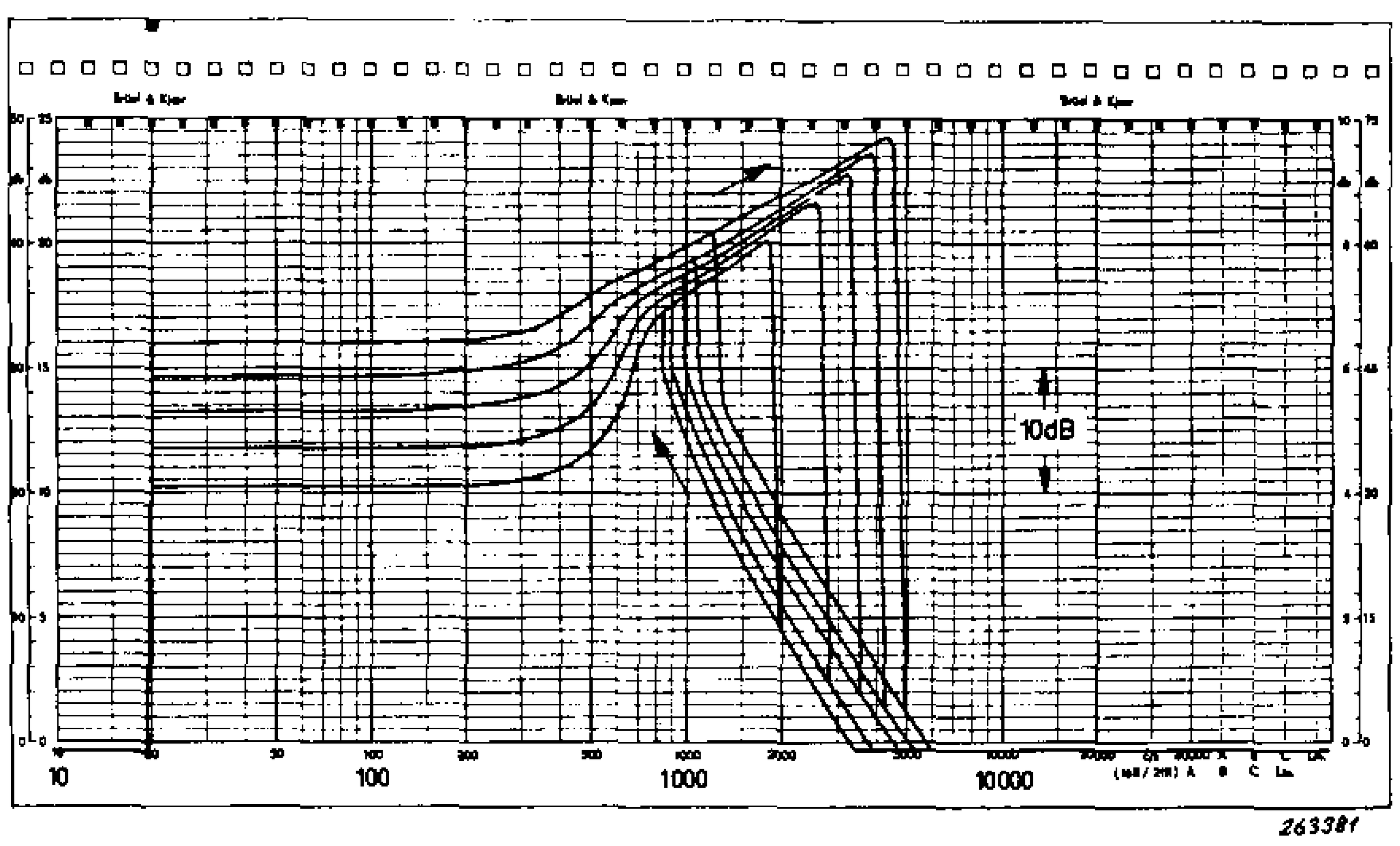

Fig. 24. Typical frequency response curves for a hardening spring system at various levels of excitation.

The remaining part of this paper will therefore be devoted to various aspects of random vibration testing, concluding with a few remarks on vibration test practice.

The Concept of Random Vibration. Wide Band Random Vibration Testing.

Random motion of a mass may be the result of a very large number of impacts, the impacts occuring by chance, and not obeying any periodic law. If the number of events is large enough the instantaneous acceleration of the mass will follow a so-called Gaussian (normal) probability density law as long as linear relationships hold true, see also Fig. 25. Due to the large "number of events" the acceleration versus time curve shown in the figure has a completely continuous character. The continuous and irregular character of the acceleration Fig. 25 are the main reasons why concepts such as "probability density" and "power spectral density" have to be introduced to give a relatively satisfactory description of the vibrations.
While the characteristics of a periodic motion are known when one period of motion is known the exact future amplitude of a random vibration signal cannot be predicted even if its complete history is known. It is, however,

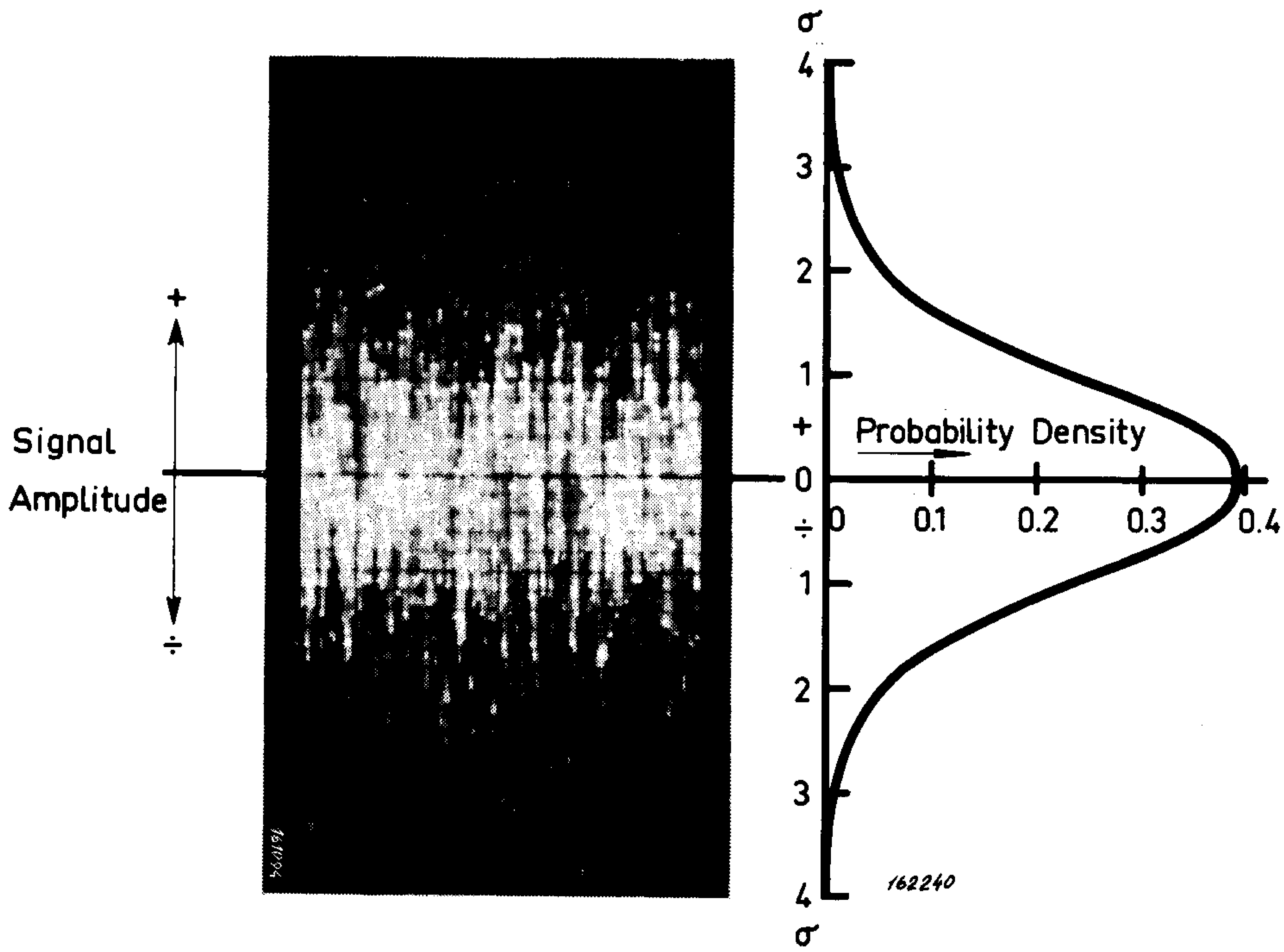

Fig. 25. Illustration of the relationship between the instantaneous magnitude values in a Gaussian random noise signal and the Gaussian probability density curve.

possible to determine the probability of finding amplitude values within a narrow amplitude window, Δ x, see Fig. 26. Now, as the probability of finding amplitude values within Δ x will depend upon the width of the window it has been found more convenient for a description to use the concept of probability density instead of probability. The probability density is found by dividing the probability of finding amplitude values within Δ x by Δ x:

$$p(x) = \lim_{\Delta x \to 0} \frac{P(x, x + \Delta x)}{\Delta x} \quad \text{where } P(x, x + \Delta x) = \frac{\Sigma_i \Delta t_n}{T}$$

If the probability density curve for a random vibration signal is found to be Gaussian, such as illustrated in Fig. 25, and the vibration system behaves linearly, all that is needed for a complete probabilistic description of the signal is its spectrum in terms of power per unit of frequency. This power spectrum is continuous, and quite analogously to the introduction of probability density an excellent description of a continuous frequency spectrum is obtained in terms of its "power spectral density":

$$G = \lim_{\Delta f \to 0} \frac{\sigma^2}{\Delta f}$$

The "window", Δ f, is in this case not an amplitude window but a frequency window, and the quantity σ^2 is the square of the RMS acceleration measured within this window, i.e. a quantity proportional to power and which is independent of any phase relationships in the (composite) signal. Very often the assumptions of linearity and Gaussian probability density are made, even when this is not explicitly stated.

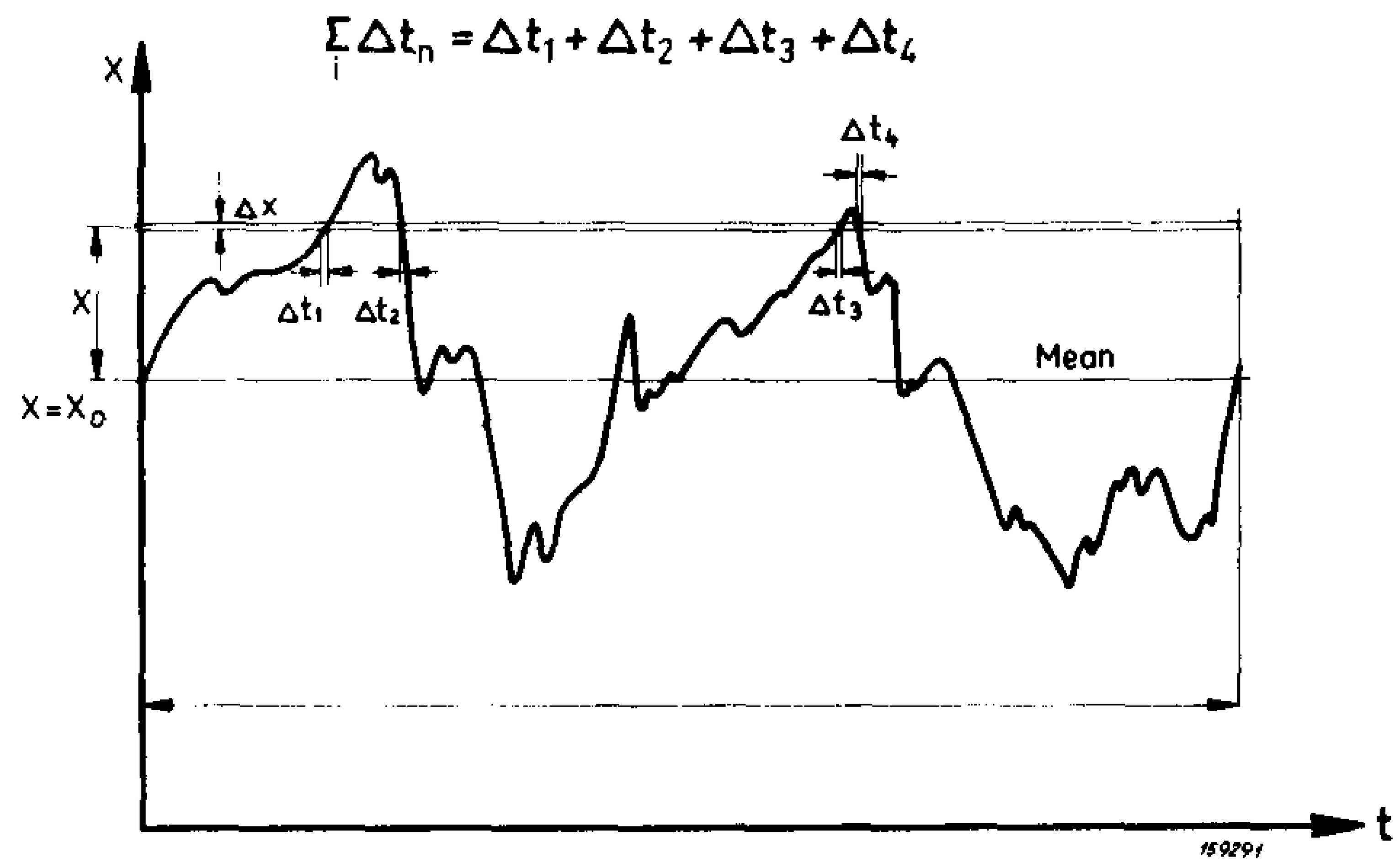

Fig. 26. Time record of a random process and illustration of the concepts of probability and probability density.

As can be seen from the above brief discussion the power spectral density is a very important concept in random vibration testing.
If a vibration machine loaded by a resonant specimen is being fed with wide-band random noise of constant power spectral density, the power spectrum of the acceleration at the specimen fixing point might look something like the curve shown on top in Fig. 27. This means that the excitation of a particular specimen resonance is different from that assumed on the basis of the vibrator input signal spectrum (flat power spectrum).
During the course of time various methods have been developed to compensate electronically for these peaks and notches. The first method is fairly straight forward and consists in using a number of "peak-notch" equalizers in cascade. Each such equalizer should exhibit the inverse response of one resonance as measured on the vibration exciter table when the system is

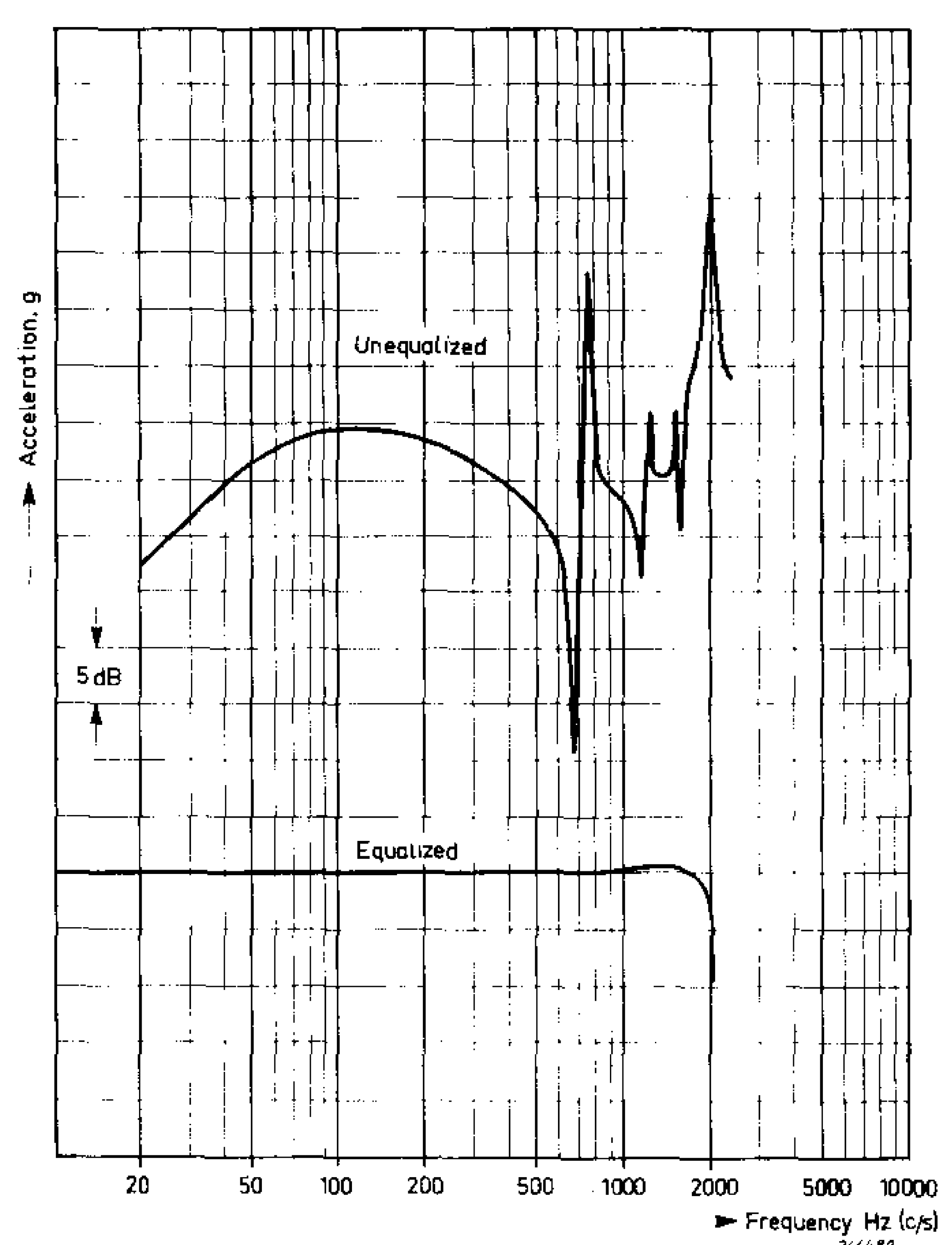

Fig. 27. Example of ideal equalization of a wide-band random vibration test system.

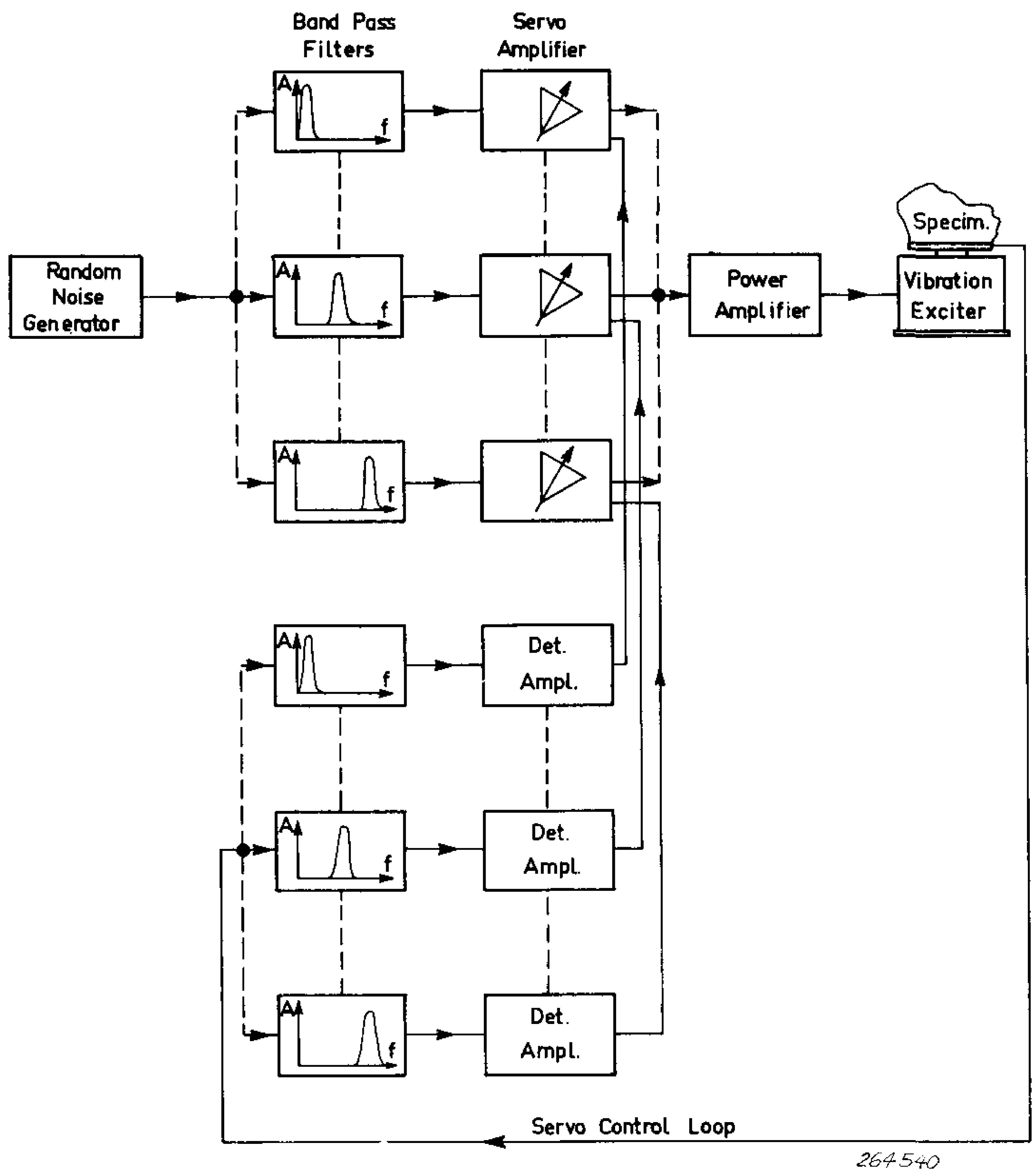

Fig. 28. Principle of operation of an automatic multiband random vibration test system (up to 80 bands are commonly used).

uncompensated. By using as many equalizers as there are resonances it is possible to achieve a response similar to that given in Fig. 27, bottom curve (equalized). This shows the resulting vibration of the shaker table when both the frequency non-linearities of the vibration machine itself and those due to specimen resonances are ideally compensated for.

The time spent in adjusting for a particular test is, however, rather great and other, more feasible methods of compensation have been sought by the producers and users of random vibration test equipment.

One such method is the so-called multiband equalization technique. It involves splitting the test frequency range into a number of discrete filter bands and adjusting the attenuation (or amplification) of each band individually. The adjustment can be performed manually or automatically. In this way it is also a relatively simple matter to "shape" the test frequency spectrum according to any given test specification or for the purpose of further test research. A disadvantage of multiband systems is that for a relatively accurate "peak-notch" compensation of complex test specimens a vast number of filters is required. Also, the setting-up time of the manual systems is quite considerable. This time is minimized in the automatic multiband systems, which on the other hand are rather complex and expensive, see also Fig. 28. *However, one great advantage of using automatic equalization is the automatic*

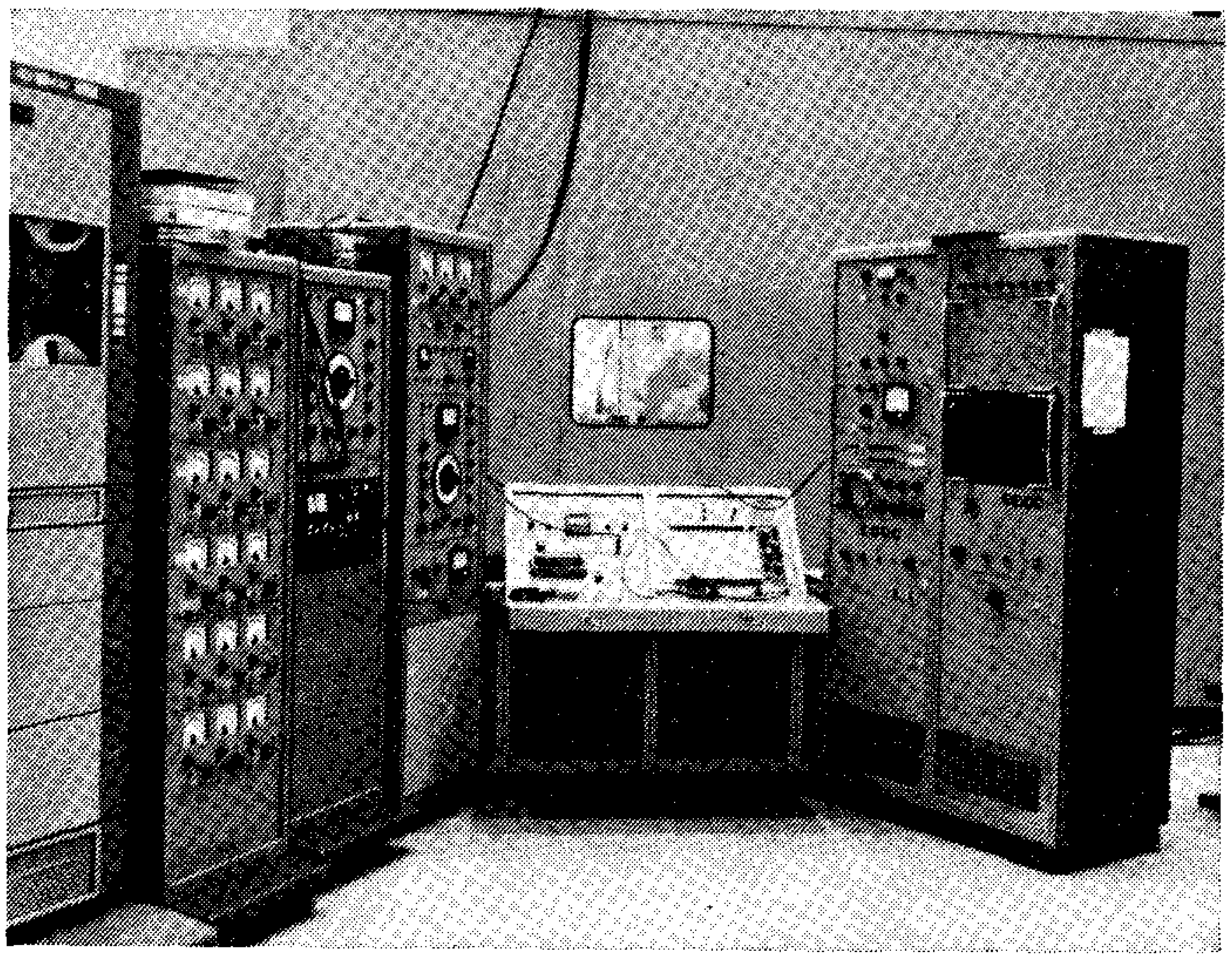

Fig. 29. Photograph of control equipment used in present day vibration testing.

correction for the effect of amplitude nonlinearities which is obtained in this case.

Fig. 29 shows a photograph of the control equipment used in wideband random vibration testing.

In the section of this paper describing the sweeping sine wave test it was mentioned that the automatic wideband random test was much less time consuming than a frequency sweep test and is thus often to be preferred when qualification testing. There are, however, also other reasons why the wideband test has become popular, and these are technically more proper than the question of testing time. The two basic ones are:

1. The vibration producing mechanisms found in nature are more of a random type than of a sinusoidal type and a random vibration test therefore simulates the statistical character of common vibration environments better than does a sine wave test.
2. The wideband random test excites all specimen resonances simultaneously so that possible interaction effects are accounted for.

Even though the vibrations commonly met in nature have a random amplitude character their power spectra vary considerably. This is taken into account (or rather *should be* taken into account) when random vibration test specifications are developed. If the same principle of enveloping is used in the development of wideband random test specification as is common practice in developing frequency sweep tests a test spectrum of the type shown in Fig. 30 normally results. However, due to structural filtering effects and "mismatching" of mechanical impedances a test spectrum developed in the way outlined above will often punish the specimen much more severely than will the actual vibration environment encountered during normal operation, Fig. 31. Also, it generally requires much larger vibration machines and power amplifiers than might otherwise be necessary.

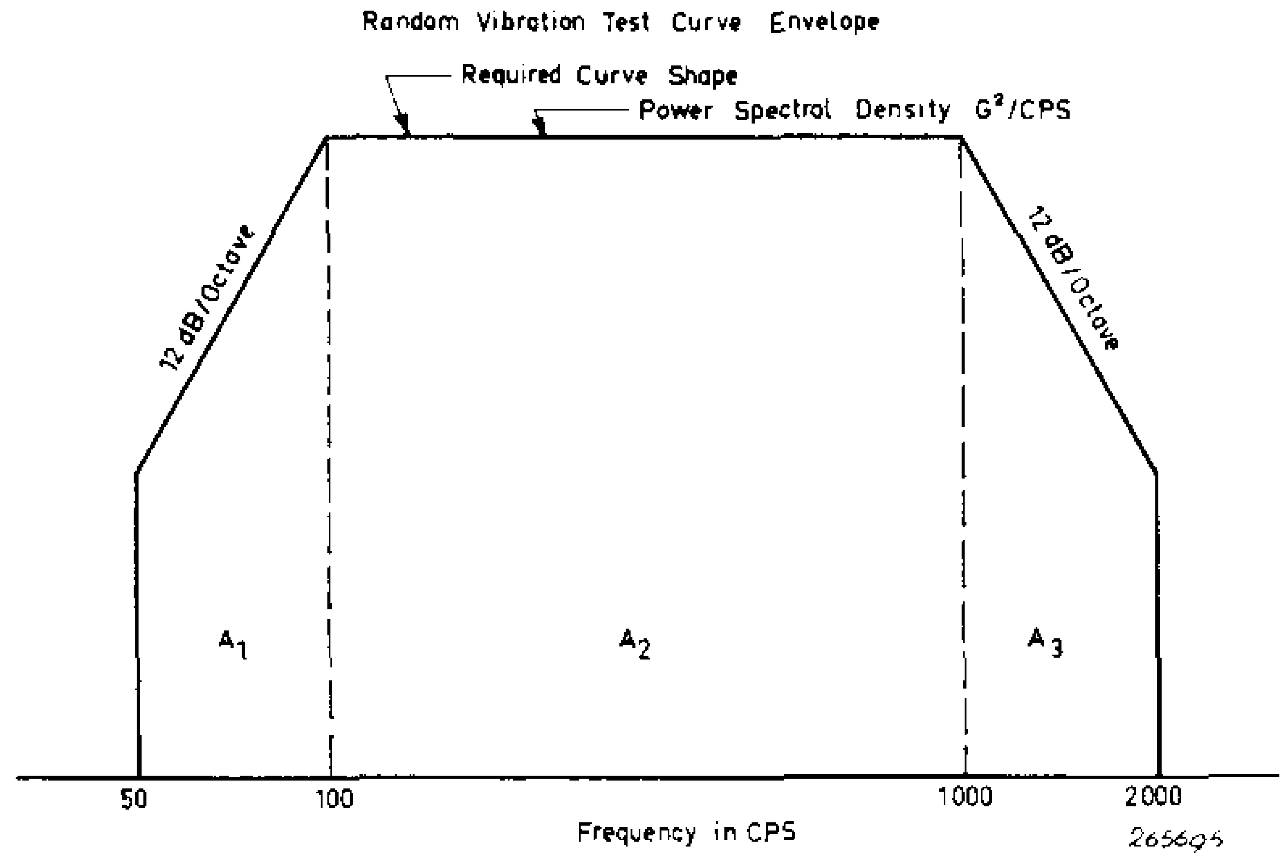

Fig. 30. Typical wide band random vibration test spectrum (MIL-STD-810 (USAF)).

In an attempt to remedy these disadvantages it is not uncommon to specify a certain wide-band "background" vibration upon which is superimposed a sweeping sine wave or a narrow band of noise. This method of *testing, however, comes close to what is known as the sweep random test technique.*

Sweep Random Vibration.

In general wide band random vibration testing is very costly and means to substitute this by a less costly test have been sought ever since its first introduction. Various "equivalent" sweep sine tests have been proposed but since the sweep sine test cannot produce the same distribution of acceleration and stress amplitudes within the test specimen no general equivalence between the two types of test is ever likely to be found.

In 1957 M. W. Olesen of the Naval Research Laboratories in the U.S.A. suggested the use of a sweeping narrow band random vibration test. This kind of testing is now commonly termed "sweep random" vibration testing. It was originally meant as an "economic" substitute for the wide band random vibration test, and the procedures for setting up the test were based on a wide band "equivalence". There are, however, other methods of setting up a sweep random test as will be discussed below.

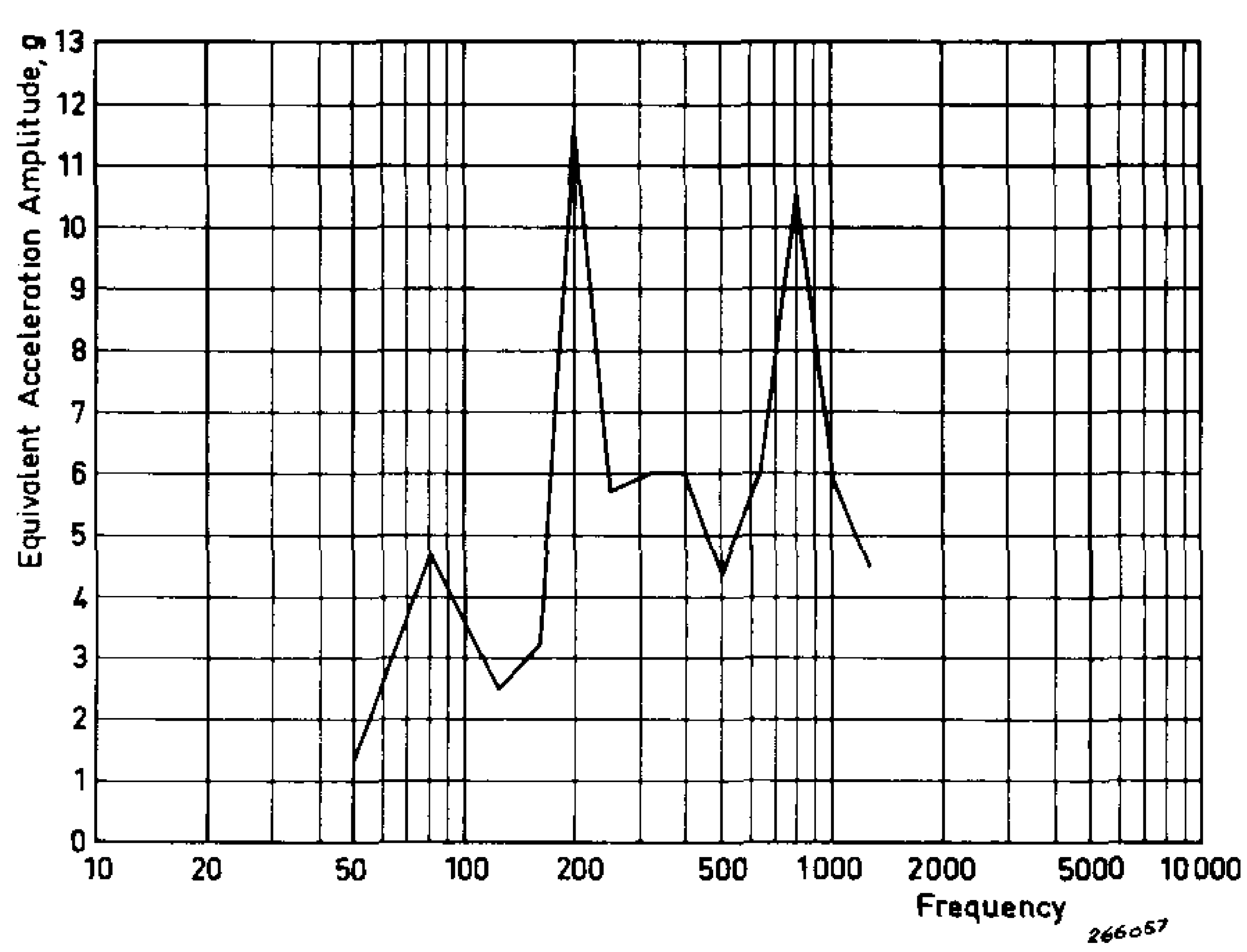

Fig. 31. Acceleration spectrum of the vibrations at a point of instrument mounting in a flight vehicle (Grede & Lunney).

There seems to be little doubt that a random vibration test simulates the characteristics of real vibration environments met in sea, road and air transport better than does any kind of pure sinusoidal testing.

This is important to consider when a vibration test is specified, as most components and sub-assemblies contain several failure mechanisms. They may contain contact mechanisms, they may be constructed of combinations of ductile and brittle materials, they may show mechanical-electrical modulation effects, etc. Many of these mechanisms will react differently to sinusoidal and random vibration excitation.

Leaving the sweeping sine-wave test as more or less a laboratory tool for investigations purposes, the only existing sweep test for practical qualification testing is the sweep random test. This test has the deficiencies that resonances are tested sequentially, and possible interaction effects must be investigated on a prototype development stage prior to production testing – and it takes a longer time than the wide band random test. Both of these deficiencies can, however, be more or less overcome by using a "multiple sweep random technique" and an accelerated test procedure.

The main advantages of the sweep random vibration test are:

1. A certain test level can be obtained by the use of much smaller power amplifiers and vibration machines than if the test was carried out as a "normal" wide band random test.
2. The statistical character of the test signal is retained.
3. The setting-up and control of the test level is simpler and in many cases more accurate than in the case of wide band random testing.
4. It is possible to start with a simple system and add complexity as required.

The original method of setting up a sweep random vibration test consisted in adjusting the test parameters so that the same number of important stresses and acceleration peaks were produced at each level as is produced by a wide band test. For a long time the equipment necessary to perform this kind of testing was not commercially available, and very little progress was thus made in the use of the method. In 1964, however, the complete control equipment required for sweep random vibration testing was developed in the form of two units: A Sine-Random Generator and a special Vibration Meter. Also considerable effort has since been made to evaluate and extend the test.

Now basically, how can the characteristics of a sweep random vibration test be adjusted to be equivalent to those of a wide-band test?

Since damage is normally caused when the test specimen resonates it has been considered important that the response of a specimen resonance is the same whether the specimen is subjected to a sweep random test or to a wide band random test. The adjustment of the sweep random vibration characteristics should thus be made so that they produce the same response at the resonance as would the wide-band test *no matter where in the test frequency range the resonant frequency of the specimen lies.*

As the bandwidth of a resonance having a specific Q-value increases with increasing frequency it was necessary to increase the sweep speed of the constant narrow band test-noise with frequency in order to obtain the same number of stress reversals inside the resonance as would result from wide band random excitation. A logarithmic sweep rate satisfies this condition, and was therefore used for the test.

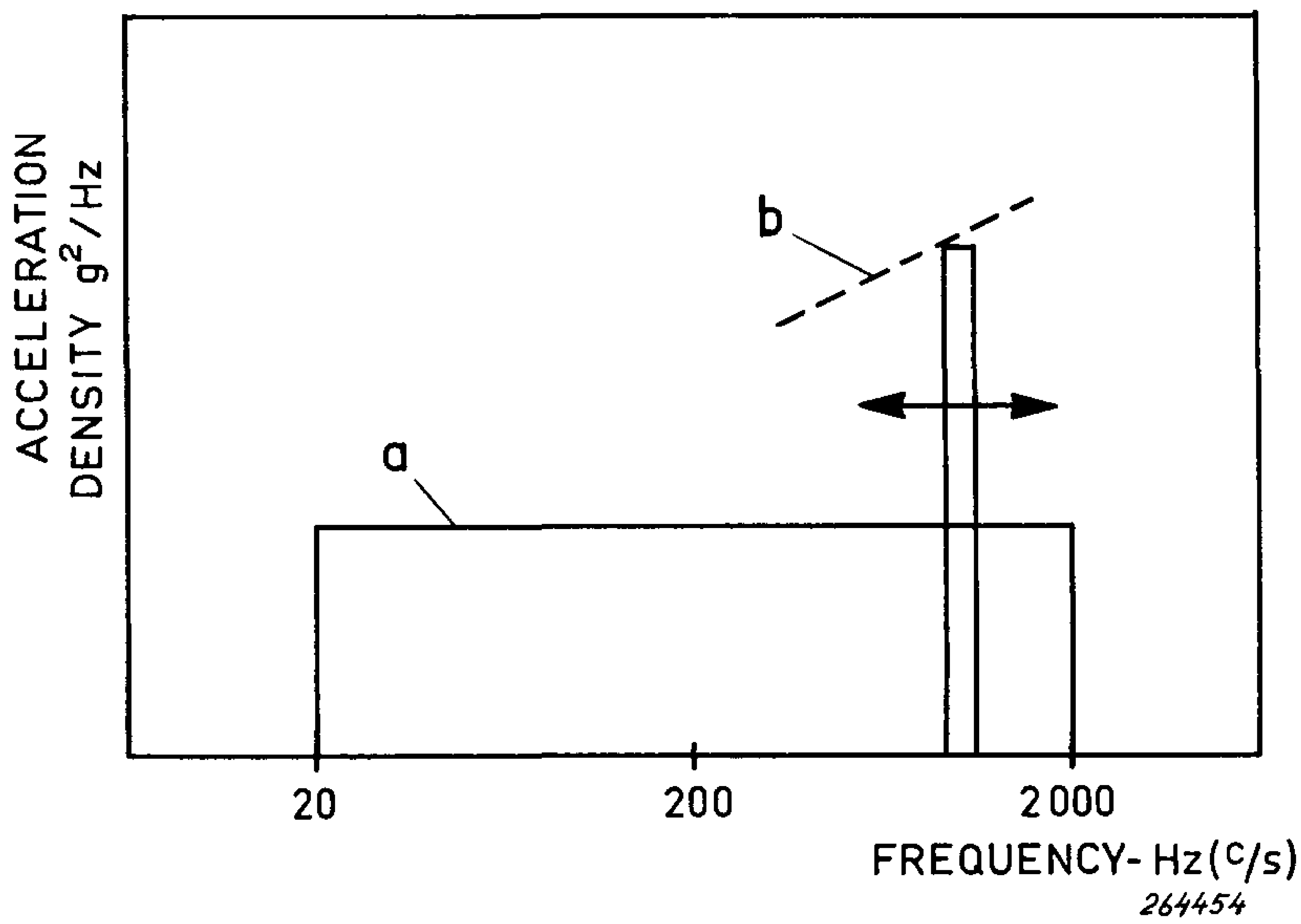

Fig. 32. Vibration excitation spectrum density:
a) Wideband random excitation.
b) Sweep random excitation.

The next problem to consider was that of obtaining an equal number of stress reversals *in any increment of stress level* for the two types of test. Again resonances of equal Q but different center frequencies were considered. The RMS response of such resonances increases with the square-root of frequency (3 dB/octave).

It was thus necessary somehow to increase the RMS value of the test band with frequency. As the bandwidth of the test band in the practical sweep random test is kept constant throughout the sweep, the amplitude values of the signal must be increased according to a 3 dB/octave law, see Fig. 32.

The increase in amplitude with frequency is taken care of automatically by circuits built into the Sine-Random Generator. It was therefore found convenient to introduce the term "acceleration gradient", the gradient here referring to "frequency space". Instead of specifying the test in the form of a varying power spectral density it could be conveniently specified in terms of a "constant acceleration gradient" ($g \times \sqrt{sec}$).

So far the total number of stress reversals as well as the RMS test level have been equated for the sweep random and the wide band random tests. However, it is required that the number of stress reversals *in any interval of stress levels* should be the same for the two tests.

This means that also the probability distribution of the peaks "around" the RMS test level should be the same in both cases. Normally the bandwidth of the test band is considerably smaller than the bandwidth of the resonance being tested and the determination of the "accumulated peak distribution" from a sweep through the resonance is a fairly complicated matter.
On the other hand, by the use of analogue models the distribution for various test conditions have been determined and compared with the "ideal" distribution. However, to better understand the problems involved in a distribution matching it is necessary to consider the response of a resonant, highly damped, single degree-of-freedom system to random vibrations.

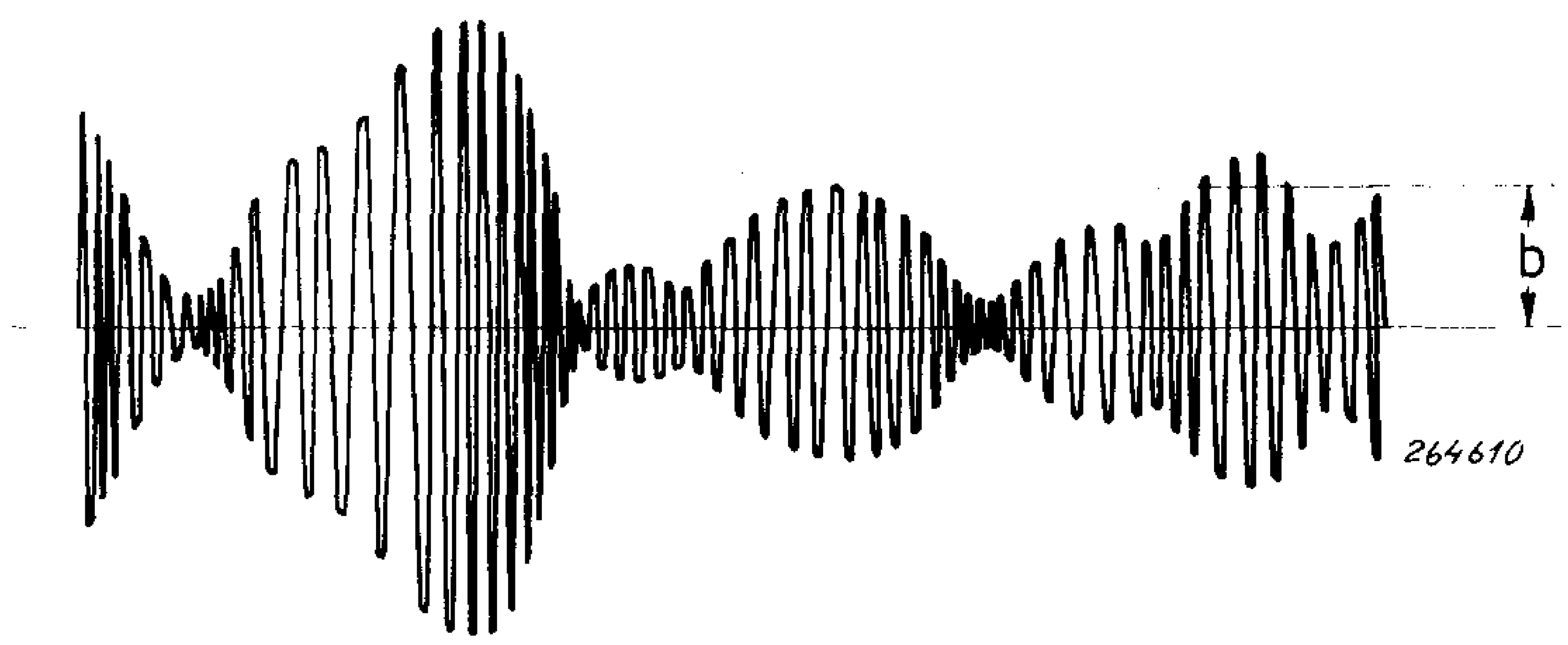

Fig. 33. Narrow band random acceleration signal with indication of the level b.

Fig. 33 shows a time record of the response of such a system to a wide band random input. (By wide band is here meant a random vibration signal with constant power spectral density over a frequency region that is much greater than the bandwidth of the resonance). It can be seen that the signal looks very much like a modulated sine wave, the frequency of the sine wave being roughly equal to the resonance frequency of the system.
By measuring the number of peaks occuring above the level b in a certain amount of time and dividing the result by the total number of peaks occuring in the same time interval, a measure is obtained for the probability of occurrence of peaks larger than b. If b now is varied from zero to ∞ a cumulative probability plot as shown in Fig. 34 can be obtained.
By sweeping a band of noise through the resonance, the bandwidth of the noise being much narrower than the bandwidth of the resonance, a similar curve for the "accumulated peak distribution" can be obtained, Fig. 35.
It is now possible to adjust the parameters of the sweeping narrow band noise so that the two curves Figs. 34 and 35 are brought to nearly cover each other, Fig. 36. This is the principle of distribution matching used to determine the exact value of the sweep random test parameters, when an "equivalence" between this and a wide band random test is sought.
Actually, the same principle is utilized when accelerated tests are designed. This will, however, not be discussed in the present paper and interested readers are referred to the Brüel & Kjær Techn. Review No. 3, 1965.
A very important feature of the sweep random vibration test technique is the possibility it offers for "simple" automatic level regulation. The use of a

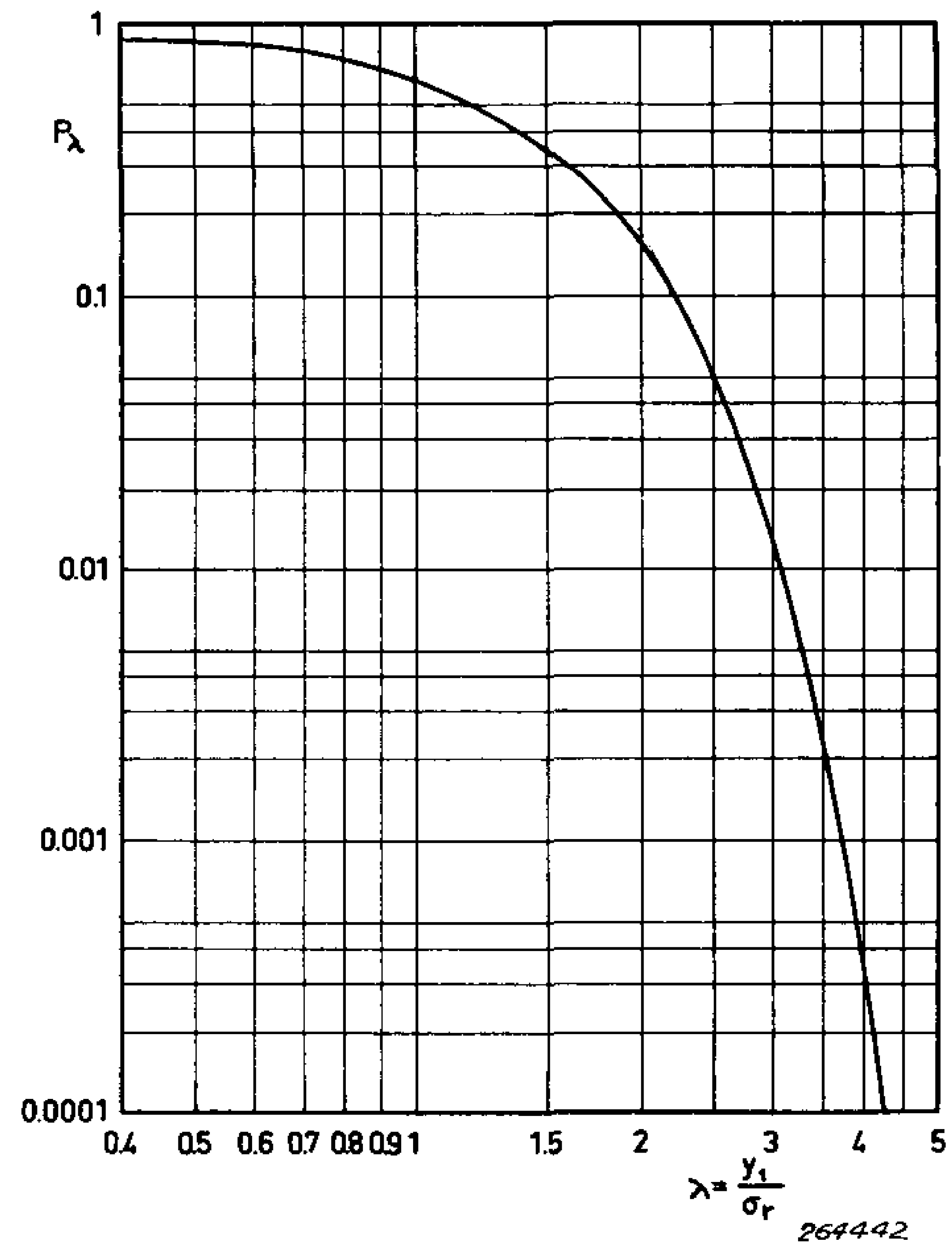

Fig. 34. Probability of occurance of peaks larger than the level b (Fig. 33) plotted to a logarithmic scale.

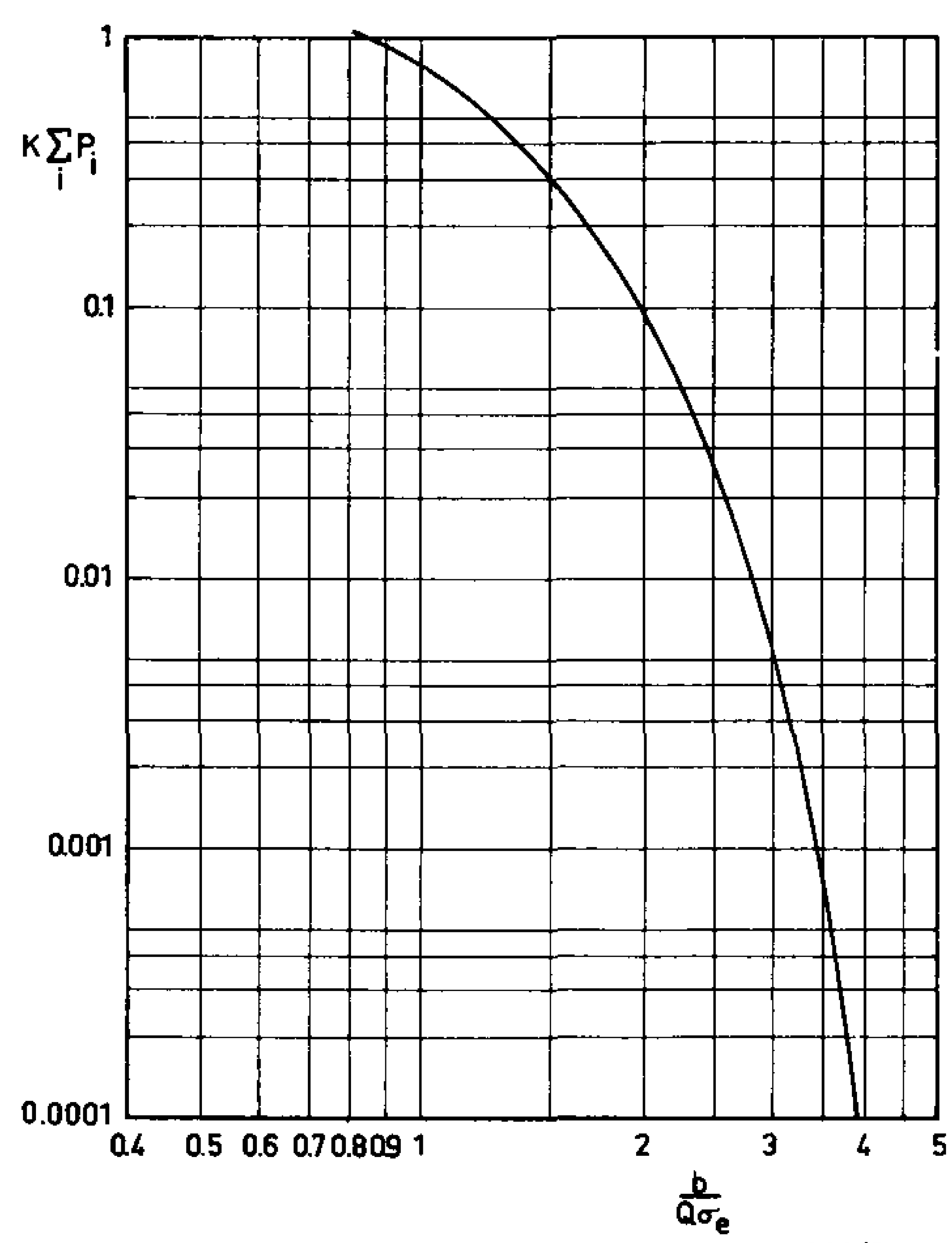

Fig. 35. "Accumulated" peak response distribution obtained by sweeping a very narrow band of noise through the resonance of the test system.

narrow band of noise as test signal makes it possible to utilize a servo regulation technique which is similar to that used in the sweeping sine wave test, Fig. 37.

Also the same programming facilities are available for the sweep random test as for the sweeping sine wave test. This together with the trend in the later years to use a sweeping signal superimposed on a low level wide band

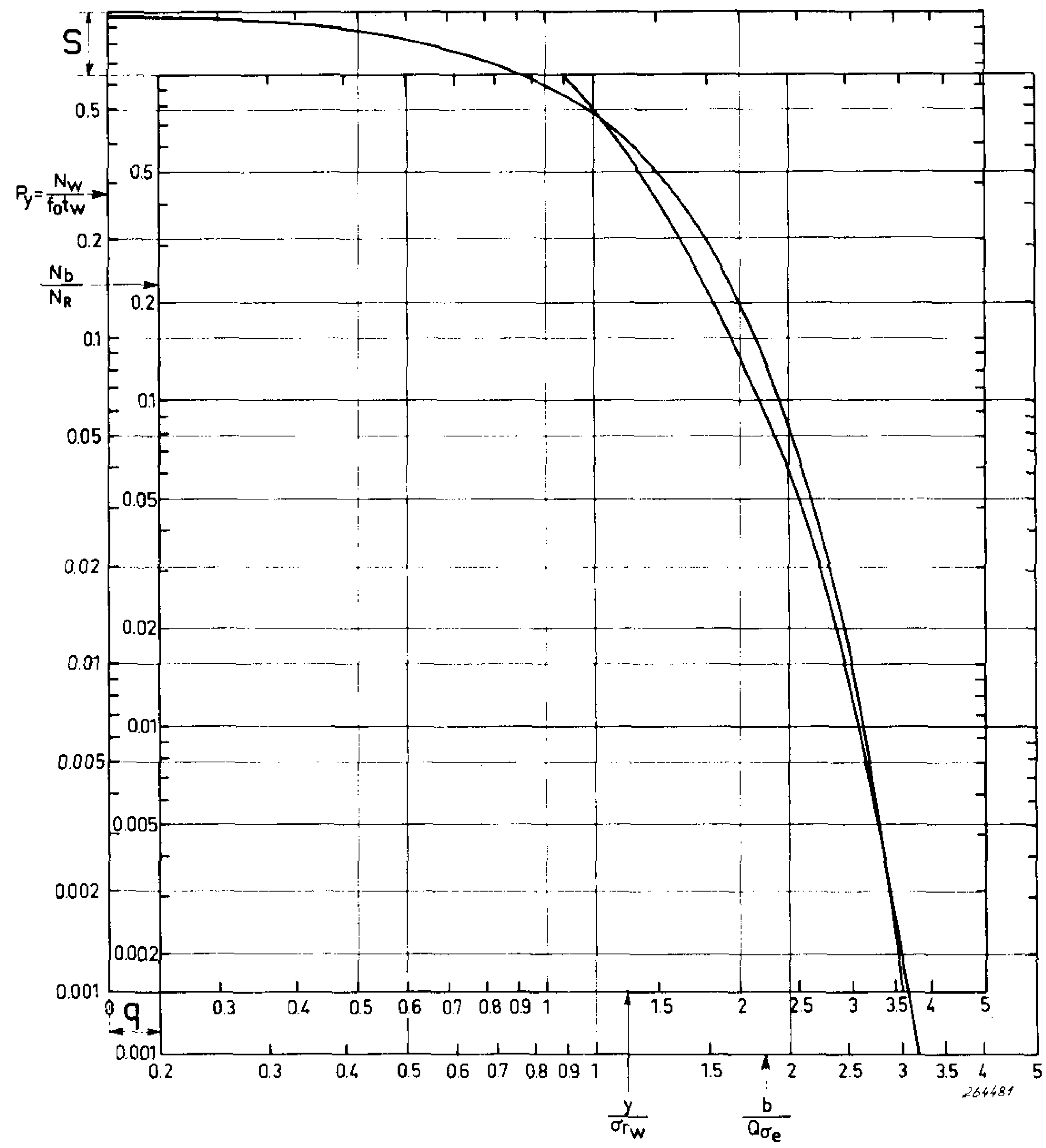

Fig. 36. Example of peak distribution matching.

background vibration seem to bring new aspects into the sweep random test philosophy. Actually it is more natural to consider the sweep random test as a separate type of testing which retains the simplicity of a sweep test and which adds the feature of a statistically well defined distribution of amplitude values than trying to establish "equivalence" with wide band tests. It is therefore suggested that sweep random test specifications should be derived in much the same way as has been usual practice for sweep sine data i.e. directly from the measured or estimated vibration environment and not as previously made by "transforming" a wide band random test specification into an "equivalent" sweep random test.

Furthermore, it seems that one or more narrow bands of noise sweeping (or dwelling) according to more detailed test programmes and emphasizing various important frequency regions would be desirable and convenient. A possible measuring arrangement for this kind of testing is shown in Fig. 38. Here one narrow band noise generator, with associated tracking filter, sweeps back and forth in one frequency region, a second in another frequency region etc. The number of narrow band generators necessary to perform the proper test will then depend upon the number of important frequency regions. In the simple case only one generator (without tracking filter) is needed. The arrangement can then be extended to include more generators whenever required.

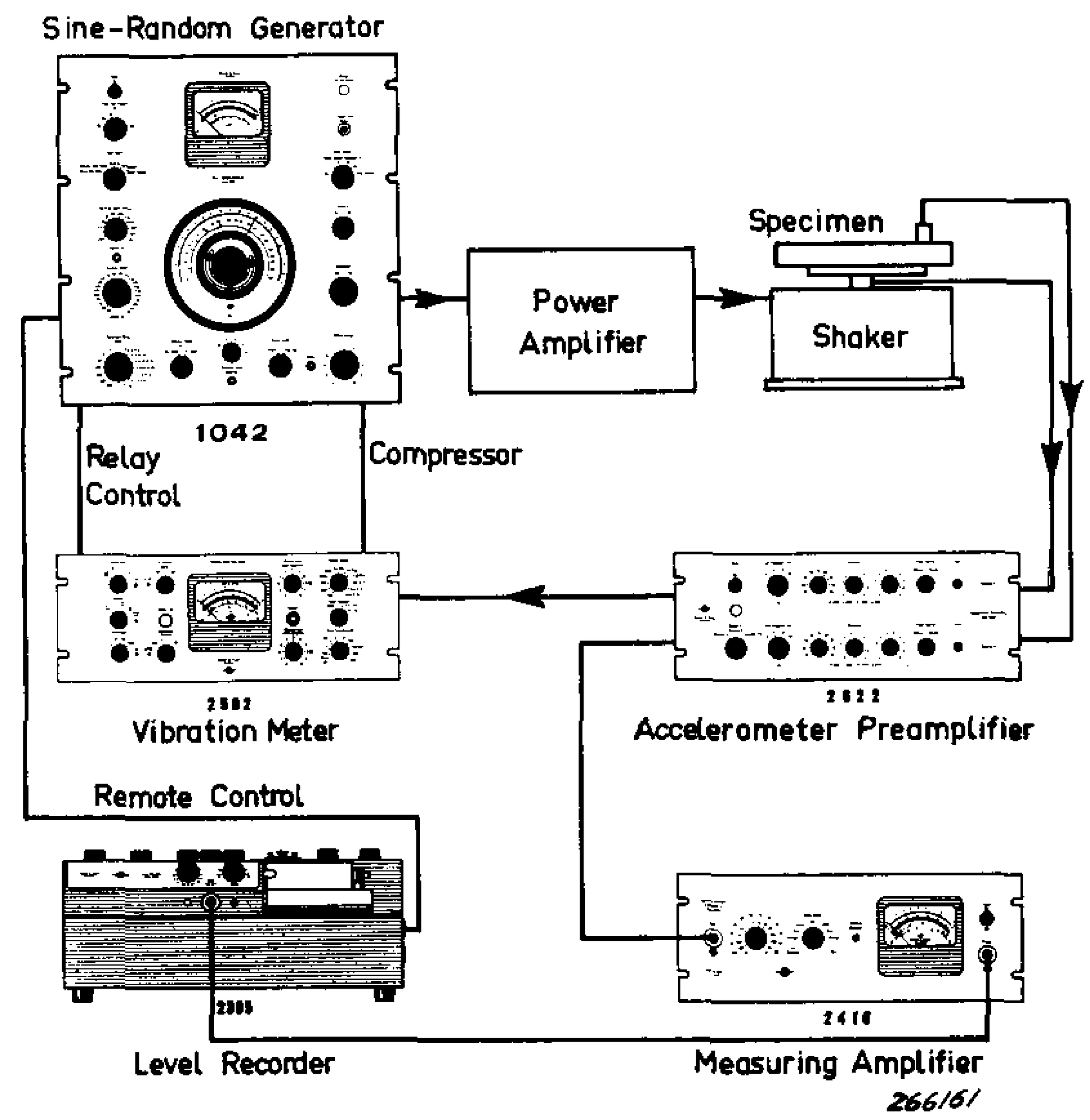

Fig. 37. A typical sweep random vibration test arrangement using a single narrow band unit only.

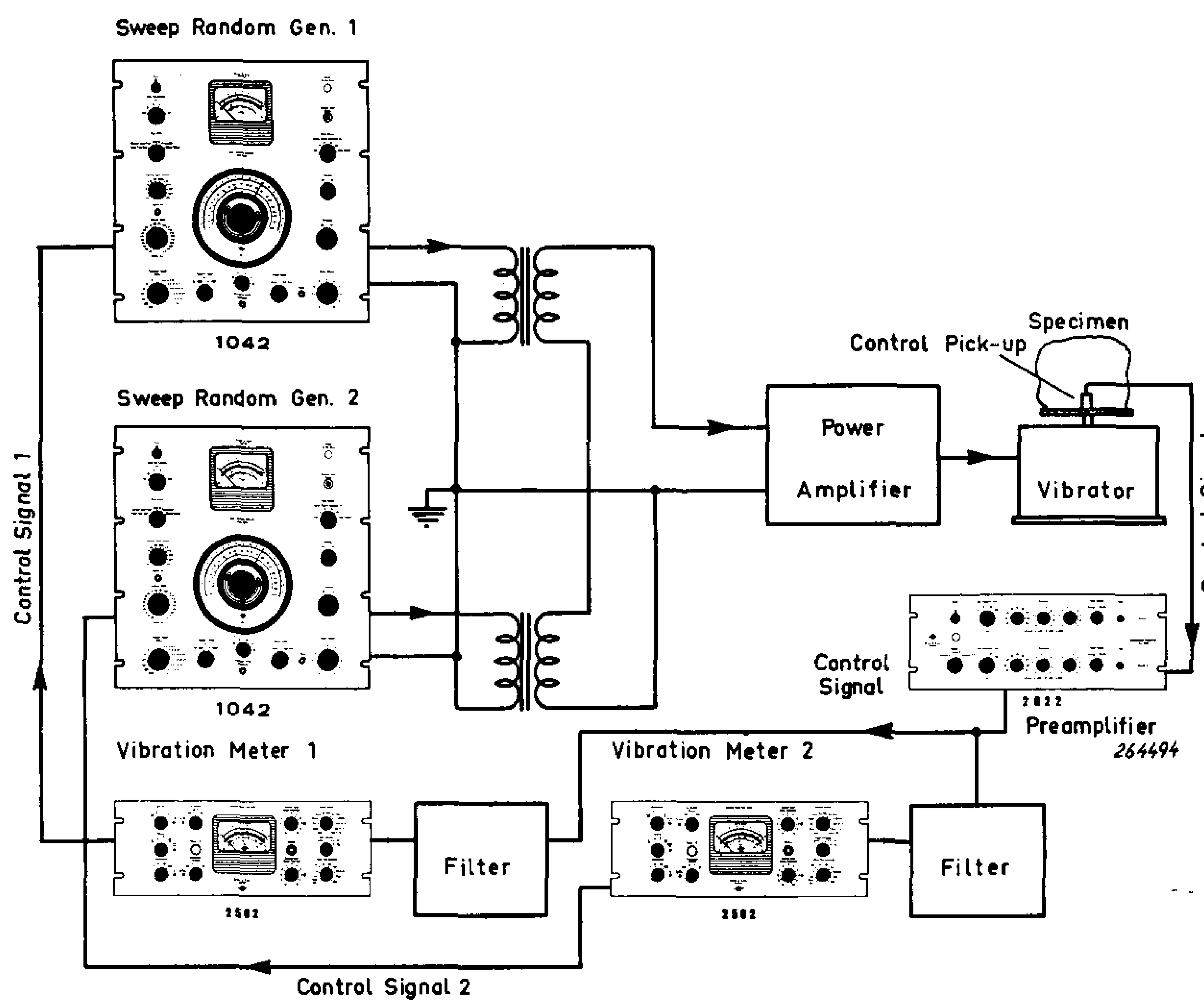

Fig. 38. Example of the use of multiple sweep random testing.

In this way the capital investment required to perform adequate random vibration testing is reduced to a minimum and a very flexible test system is obtained.

Conclusion.

In the preceding text some of the most important vibration test methods in use to-day as well as outlines of the philosophies underlying the tests have been given. There are, however, many factors involved in practical vibration testing which have not been considered here and a more thorough discussion of these factors is deemed to be outside the scope of this paper. On the other hand, it would seem appropriate to conclude the discussion by mentioning but a few of these factors and touch upon some of the future outlooks in the field of vibration testing.

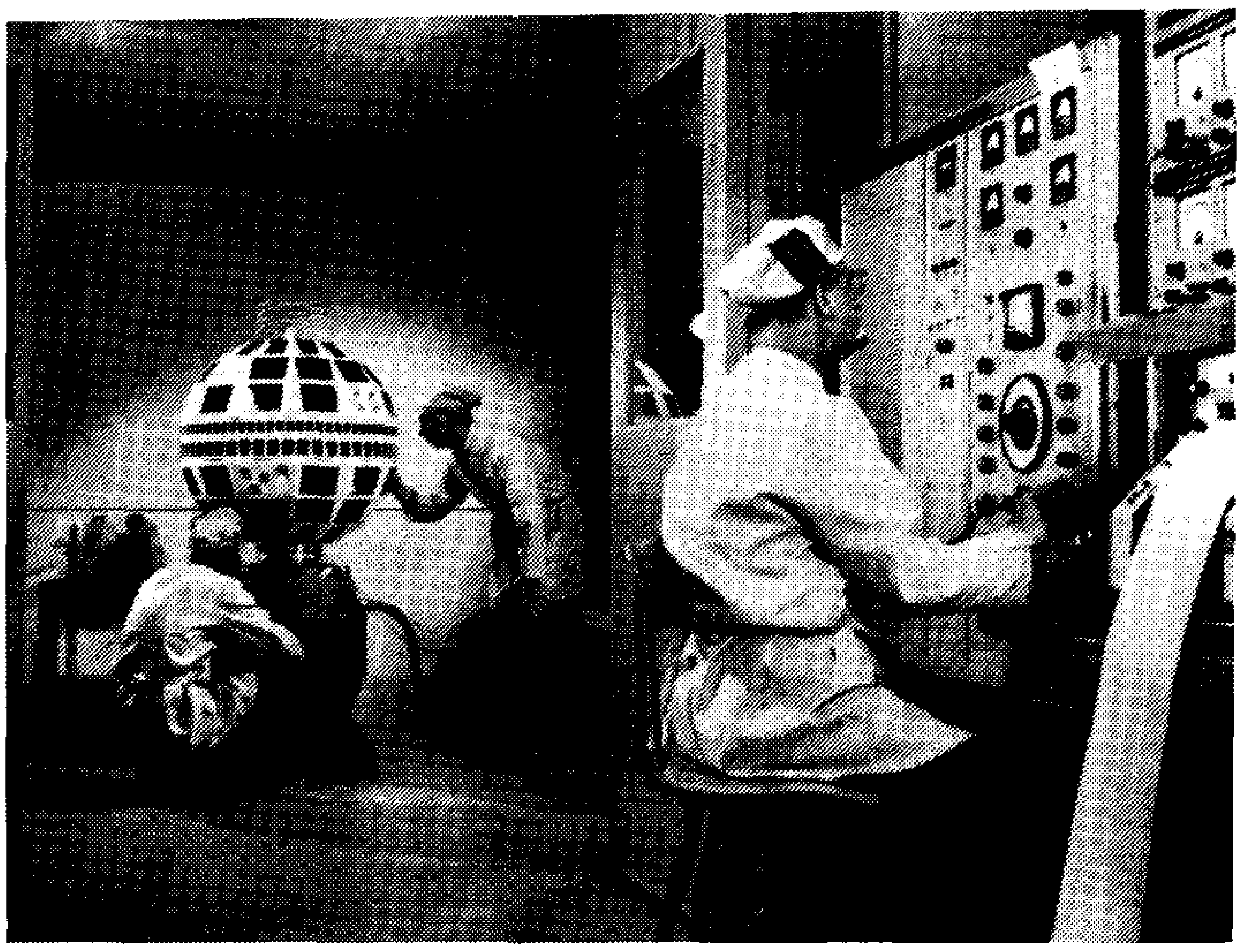

Fig. 39. Testing of a satelite in a combined environmental test chamber.

An important factor to be considered in designing a practical vibration test is the development of fixtures and guides to be used in attaching the test specimen to the vibration machine. These should be as rigid as possible and preferably no resonant mode should be present within the frequency range of the test. In testing large specimens use is often made of so-called "slippery tables" which act both as guides and as supports for the test specimen.

A guiding system is also often required to meet the specification requirements for rectilinear vibration of the specimen in each of three mutually perpendicular directions, with a minimum of rocking and other multidirectional effects. Another requirement also sometimes called for in vibration test specifications is that the test should be performed under particular atmospheric conditions, which means that the vibration machine might have to be placed in special environmental chambers, Fig. 39. This requirement may be most common in cases where the test specimen consists of a satellite or other space investigating equipment.

When a sweeping sine wave test is specified a very common requirement is that of limited harmonic distortion of the control signal (feedback signal). To meet this requirement a tracking filter must often be inserted in the feedback control loop. The new IEC Recommendation 68/2/6 for sweeping sine wave tests, for instance, presupposes the use of a tracking filter when the distortion of the control signal is larger than 25 %.

Before carrying out a practical vibration test according to a prescribed test specification, correct operation of each part of the instrumentation, including

the vibration machine and the fixture used for the test, should be checked.
After having mounted the test specimen it is, furthermore, common practice to run a check-test at very low vibration level, thus making sure that the complete arrangement operates properly. The vibration level is then increased to that prescribed by the specification and the actual testing begins. To ensure shutdown of the test in case of instrumentation failure both the vibrator and the power amplifier are often supplied with overload protection devices.
So far as to practical vibration testing. Even though the use of electrodynamic vibration machines may be considered the most popular vibration generation method at present, another very different method seems to have gained considerable importance in later years: the method of acoustically induced vibrations.

Several kinds of sound generation are used, and the test consists in exposing the specimen to an intense noise field. Sound pressure levels in the regions of 140 to 170 dB re 2×10^{-5} N/m^2 are desirable and not uncommon. This method of testing requires, of course, very large capital investments due to the poor efficiency involved in the vibrational energy transfer, but it is deemed in many cases to be a more realistic test than the one obtained by using electro-mechanical vibrators.

One problem, which is avoided by the use of sound induced vibrations, and which is at present being studied by various research institutions is the influence of mechanical impedance upon the outcome of common vibration tests. As will be clear from the discussion in this paper vibration tests are normally specified in terms of motional quantities such as acceleration, velocity or displacement, taking little notice of the force producing this motion. As a motion-controlled vibration machine may be regarded to have an infinite mechanical impedance*) and normal structural vibration generators, such as vibrating panels and beams, have very finite mechanical impedances the appropriateness of the simulation technique used in to-day's vibration testing might be questioned. Attempts have therefore been made to introduce force control of the vibration machine instead of motion control. As the technique and knowledge in the field become greater, some sort of combined motion and force control seems to be a relatively reasonable solution, but considerably more research needs to be done before a firm answer can be given to the above problem.

Finally, it should be mentioned that the spread in vibrational behaviour of ordinary production items requires that vibration testing is performed not only on a few selected specimens. If a more general statement of the product's behaviour in a vibration environment has to be made, a rather large number of representative specimens have to be tested. This is a fact which is sometimes overlooked by a not too experienced test crew, a fact which can actually not be overemphasized.

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*) Mechanical impedance is normally defined as: $Z_{Mech.} = \frac{\text{Velocity}}{\text{Force}}$

Because so many parameters are involved in the derivation of vibration test specifications as well as in carrying out the test itself, vibration testing has sometimes been called an "art" and not a science. Whether considered an "art" or a science one thing remains however: To properly analyze the results of a vibration test *always* requires a great deal of engineering judgement.

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